AOE POE Integrated Curriculum Unit: Bombs Away

Lesson 1.1
Principles of Engineering
Ballistic Bullseye

In this lesson, students are introduced to the integrated unit and the concepts that will be covered in their various classes. The lesson begins by introducing students to the concept of **ballistic motion**, and they learn to identify examples and its applications in the real world. This lesson should take place at the start of POE Unit 8 on Kinematics.

This lesson is expected to take one class period.

Lesson Framework

Learning Objectives

Each student will:

- Describe the unique characteristics of ballistic motion, and how it differs from other types of motion.
- Identify examples of ballistic motion.

Academic Standards

- Students should develop an understanding of motion and forces. (NRC National Science Education Standards, Physical Science)

Assessment

Assessment Product	Assessment Criteria
List of real-world examples of projectile motion and their initial forces.	None provided

Prerequisites

- Working knowledge of force

Instructional Materials

Teacher Resources

- None

Student Resources

- None

Equipment and Supplies

- Computer with LCD projector

Lesson Steps

Step	Min.	Activity
		CLASS PERIOD 1
1	**35**	**Class Discussion: Projectile Motion** Download a video of a ski jumper from the Internet and show it to the class. A promo video can be found at **http://sports.espn.go.com/broadband/video/videopage?videoId=3378929&categoryId=3286128&n8pe6c=2**

Step	Min.	Activity
		(You can alter the problem to fit skateboarding, car jumping, or any other example of parabolic motion you think might appeal to your students.) Ask students to consider what they would do if they had been asked to design a 120-meter ski jump. Say that a hill with a 50-degree slope has been found that they can use. The end of the jump should launch the jumper at an angle of 15 degrees above the horizontal. The average speed of the skiers at the end of other ramp is usually about 10 meters per second. What kinds of things would they need to know? After students have discussed the problems for a few minutes, explain that the study of this type of problem is known as **kinematics**. Define kinematics as the study of the description of motion. The type of motion being described in the ski jump problem is known as **projectile motion**. Any object that is traveling through the air, as the result of an initial force only is known as a **projectile**. Based on this definition, have students identify which objects from the following list are projectiles, and for those that are projectiles, what was the initial force? • Football • Airplane • Arrow • Bird Some basic understanding of mathematics and science allows us to predict the path of projectiles if we have information about the starting conditions: for example, how much force is applied at what angle? It is easy to calculate this path if the force of gravity remains constant and we ignore the effects of air resistance. The study of projectile motion is a specialized field known as **ballistics.** Students may have heard the term “ballistics” used in association with solving crimes. You might ask students if they’ve heard the term before, and what it meant in that context. Ask students to explain how the two uses of the word are related.
2	**15**	**Direct Instruction: Integrated Unit Overview** Explain to students that they will be working on an integrated unit designed around Principles of Engineering lessons that address kinematics. As with Bridge Builder (if they have already completed that unit), their other academic classes will also contain teaching lessons associated with kinematics. Over the course of the integrated unit, students will learn about the mathematics and science principles that underlie the study of projectile motion, as well as some of important historical events that occurred as a result of the understanding of kinematics. The unit will culminate with students designing their own ballistic devices and using them in a battleship-type competition. Create and display a timeline of when the lessons of the unit will take place over the next few weeks. For homework, have students create a list of as many real-world examples of projectile motion that they can think of or find. Encourage them to bring in pictures, if possible. In the next class period, go over the examples with the class and have them identify the initial force for each example.

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Lesson 1.1

Ballistic Bullseye

Student Resources

There are no student resources for this lesson.

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Lesson 1.1

Ballistic Bullseye

Teacher Resources

There are no teacher resources for this lesson.

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Lesson 1.2

Physics or Principles of Engineering

Trajectory Motion

In this lesson, students will learn that trajectory motion is an object moving in a two-dimensional plane and that the movement takes place along a parabolic path that can be broken down into two components, vertical and horizontal. The velocity remains constant in the *x* direction, but the velocity in the *y* direction changes with time in both magnitude and direction due to the force of gravity.

This lesson is expected to take up to two class periods.

Lesson Framework

Learning Objectives

Each student will:

- Explain how velocity and acceleration are calculated.
- Calculate range and initial acceleration from data they record from experiments.

Academic Standards

- Students should develop an understanding of motions and forces. (NRC National Science Education Standards, Physical Science)
- Use symbolic forms to represent and analyze mathematical situations and structures. (NCTM Standards for School Mathematics, Algebra)
- Use mathematical models and analyze change in both real and abstract contexts. (NCTM Standards for School Mathematics, Algebra)

Assessment

Assessment Product	Assessment Criteria
Example projectile motion problems	Answers dependent on teacher–assigned variables

Prerequisites

- Working knowledge of freefall and linear motion
- Working knowledge of gravitational acceleration
- Working knowledge of how to find the sine, cosine, and tangent of an angle using a calculator

Instructional Materials

Teacher Resources

- Teacher Resource 1.2.1—Presentation: Kinematics

Student Resources

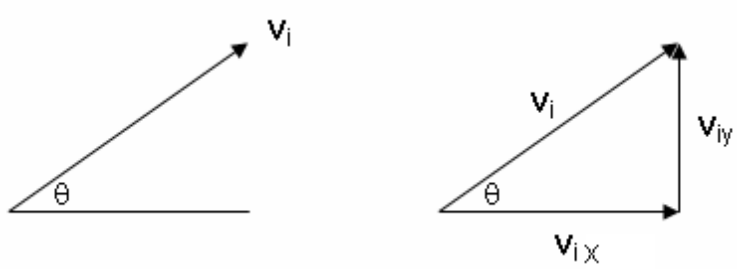
- Student Resource 1.2.1—Handout: Projectile Motion Variables

Equipment and Supplies

- Computer with LCD projector ***or*** chalkboard/whiteboard

Lesson Steps

Step	Min.	Activity
		CLASS PERIOD 1
1	**15**	**Variable Review: Projectile Motion** Pass out the Projectile Motion Variables handout (Student Resource 1.2.1). Select a projectile that will be of interest to your student population (e.g., ski, football, basketball, baseball, bullet, rocket, etc.). Draw an example on the board. Using that example, go through the list of variables from the handout and identify the variable

Step	Min.	Activity
		as part of the example. Then draw another example on the board and have students identify the variables for themselves.
2	**varies**	**Direct Instruction: Kinematics** Using the Kinematics presentation (Teacher Resource 1.2.1) as guide, review the concept of **linear motion** and introduce the concept of **trajectory motion** to students, covering at a minimum the following material: **Step 1:** Generating equations of motion for the horizontal and vertical parts of projectile motion. A general equation that is useful in the study of **projectile motion** reveals that the generalized displacement, *s*, is equal to the sum of two parts: $s = (1) + (2)$ Where part (1) is the initial velocity of the object in the same direction as the displacement multiplied by the time (the constant velocity portion of the equation), and part (2) represents one-half multiplied by the acceleration in the same direction as the displacement multiplied by time squared (the accelerated motion part of the equation). Written as a useful equation: $s = v_{is}t + \frac{1}{2}at^2$ Projectile motion is most easily understood by separating the horizontal and vertical parts of the motion. When the equation, above, is rewritten for the horizontal and vertical parts of motion, two equations emerge: Horizontal motion: $x = v_{ix}t + \frac{1}{2}a_x t^2$ Vertical motion: $y = v_{iy}t + \frac{1}{2}a_y t^2$ **Step 2:** Solving for v_{ix} and v_{iy}. We can find v_{ix} and v_{iy} in terms of v_i and θ by using trigonometry. Examining the initial velocity of a projectile at some angle, θ, we see that: 

Step	Min.	Activity

Trigonometry can provide a relationship between v_{ix}, v_i, and θ using cosine, since:

$$\cos\theta = \frac{adjacent}{hypotenuse} \text{ hence, } \cos\theta = \frac{v_{ix}}{v_i}$$

Rearranging and solving for v_{ix}, we see the following relationship:

$$v_{ix} = v_i \cos\theta$$

Trigonometry also can provide a relationship between v_{iy}, v_i, and θ using sine, since:

$$\sin\theta = \frac{opposite}{hypotenuse} \text{ hence, } \sin\theta = \frac{v_{iy}}{v_i}$$

Rearranging and solving for v_{iy}, we see the following relationship:

$$v_{iy} = v_i \sin\theta$$

Step 3: Simplify the horizontal and vertical equations of motion and substitute the equations for v_{ix} and v_{iy}.

Horizontal Part of Projectile Motion	**Vertical** Part of Projectile Motion
Original equation: $x = v_{ix}t + \frac{1}{2}a_x t^2$	Original equation: $y = v_{iy}t + \frac{1}{2}a_y t^2$
Applying concepts and simplifying the equation: **Concept:** Horizontal motion is constant velocity motion where acceleration = 0 Resulting Simplification: $x = v_{ix}t$	Applying concepts and simplifying the equation: **Concept:** Vertical motion is accelerated and due to gravity, so $a_y = g$ *where g is a negative value as towards the earth*. For the entire motion, the overall vertical displacement, y, is zero. Resulting Simplification: $0 = v_{iy}t + \frac{1}{2}gt^2$ or $v_{iy}t + \frac{1}{2}gt^2$
Substituting v_{ix}: Since $v_{ix} = v_i \cos\theta$ $x = v_i \cos\theta t$	**Substituting v_{iy}:** Since $v_{iy} = v_i \sin\theta$ $v_i \sin\theta t = -\frac{1}{2}gt^2$

Step	Min.	Activity
		Step 4: We want to be able to determine the velocity of the projectile given the gravitational acceleration (a constant), angle θ, and range of motion (maximum horizontal displacement). To do this, we will solve the vertical equation for v_i by dividing both sides by $\sin\theta t$ and simplifying. $v_i = \frac{-gt}{2\sin\theta t}$ **Step 5:** Because time is difficult to measure accurately for projectile motion experiments, we will use the horizontal motion equation to solve for time and then substitute this equation for time into the equation, above. $t = \frac{x}{v_i \cos\theta}$ Substituting this equation into the equation for v_i: $v_i = \frac{-gx}{2v_i \cos\theta \sin\theta}$ Simplifying this equation yields the following: $v_i^2 = \frac{-gx}{2\cos\theta \sin\theta}$ Solving for v_i yields: $v_i = \sqrt{\frac{-gx}{2\cos\theta \sin\theta}}$ Finally, using the trigonometric identity $2\sin\theta \cos\theta = \sin 2\theta$, we can simplify this equation: $v_i = \sqrt{\frac{-gx}{\sin 2\theta}}$
3	**15**	**Problem Solving: Initial Velocity** Go back to the examples you presented at the beginning of class. Assign values for *theta* and initial velocity and have students calculate the initial horizontal and vertical velocity for each of the examples for homework.

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Lesson 1.2

Trajectory Motion

Student Resources

Resource	Description
Student Resource 1.2.1	Handout: Projectile Motion Variables

Student Resource 1.2.1

Handout: Projectile Motion Variables

Variable	Variable Name	Description	English Units	Metric Units
s	Displacement	How far an object is from where it started (generalized term)	Feet (ft)	Meters (m)
x	Horizontal displacement	How far, horizontally, an object is from where it started	Feet (ft)	Meters (m)
y	Vertical displacement	How far, vertically, an object is from where it started	Feet (ft)	Meters (m)
t	Time	Time an object is in motion	Seconds (s)	Seconds (s)
v_i	Initial velocity	How fast an object moves in its initial position	Feet per second (ft/s)	Meters per second (m/s)
q	Theta	The angle the initial velocity makes with the horizontal axis (between 0° and 90°)	Degrees (°)	Degrees (°)
v_{ix}	Initial horizontal velocity	How fast an object moves horizontally in its initial position	Feet per second (ft/s)	Meters per second (m/s)
v_{iy}	Initial vertical velocity	How fast an object moves vertically in its initial position	Feet per second (ft/s)	Meters per second (m/s)
a	Acceleration	How quickly an object changes its velocity because a net force acts on the object	Feet per second squared (ft/s^2)	Meters per second squared (m/s^2)
a_x	Horizontal acceleration	How quickly an object changes its velocity horizontally because a net force acts on the object	Feet per second squared (ft/s^2)	Meters per second squared (m/s^2)
a_y	Vertical acceleration	How quickly an object changes its velocity vertically because a net force acts on the object	Feet per second squared (ft/s^2)	Meters per second squared (m/s^2)
g	Acceleration due to gravity	How quickly an object changes its velocity because of the force of gravity	Feet per second squared (ft/s^2)	Meters per second squared (m/s^2)

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Lesson 1.2

Trajectory Motion

Teacher Resources

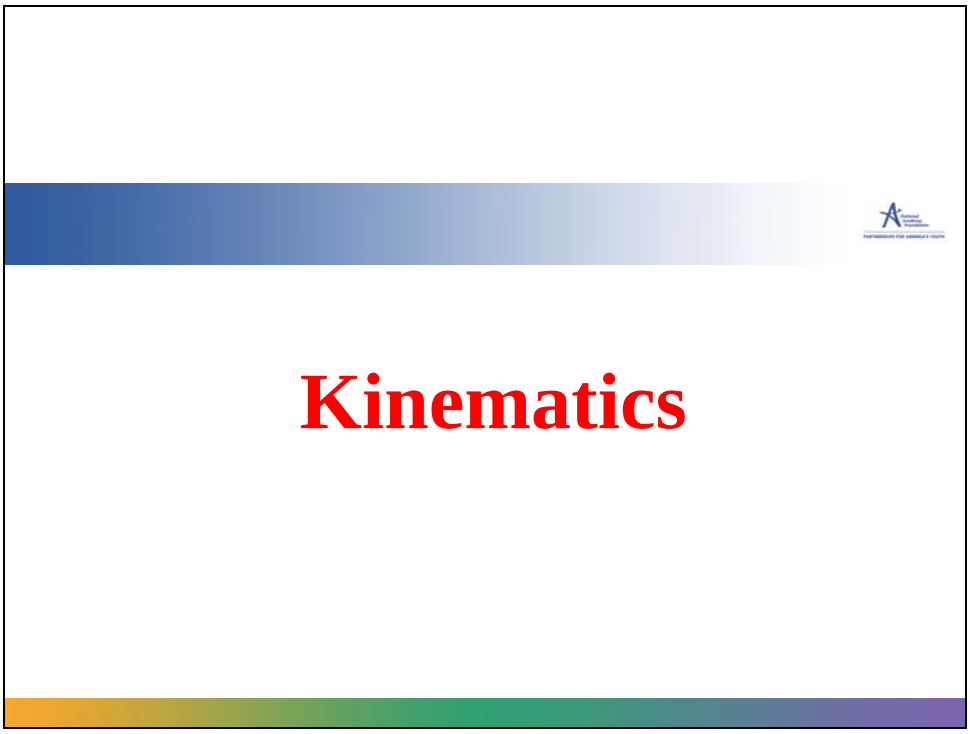
Resource	Description
Teacher Resource 1.2.1	Presentation: Kinematics

Teacher Resource 1.2.1

Presentation: Kinematics

Directions

In order to open the presentation, right-click on the image below, then select Presentation Object/Open from the menu.



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Lesson 1.3
Algebra/Geometry
"Qannon" Quadratics

In this lesson, students will apply knowledge of angles, trigonometric functions, and linear and quadratic equations to solve projectile motion problems. Students will be given a variety of scenarios where they must apply the formulas for projectile motion to determine angle, distance, or initial force at launch in order to hit a target with given parameters. Students will begin by practicing some problems as a class and then pair off to compete against each other in solving equations.

This lesson is expected to take two to three periods.

Lesson Framework

Learning Objectives

Each student will:

- Solve for one variable using the linear equations $x = v_{0x}t + x_0$ and $v_y = at + v_{0y}$.
- Solve for one variable using the quadratic equation $y = \frac{1}{2}at^2 + v_0t + y_0$.
- Represent word problems with systems of vectors.
- Apply trigonometric properties to sum component vectors.
- Apply trigonometric properties to solve for angles.
- Sketch pictures of word problems involving motion in two dimensions.

Academic Standards

- Understand patterns, relations, and functions. (NCTM Standard for School Mathematics, Algebra Standard)
- Represent and analyze mathematical situations and structures using algebraic symbols. (NCTM Standard for School Mathematics, Algebra Standard)
- Use mathematical models to represent and understand quantitative relationships. (NCTM Standard for School Mathematics, Algebra Standard)
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. (NCTM Standard for School Mathematics, Geometry Standard)

- Apply transformations and use symmetry to analyze mathematical situations. (NCTM Standard for School Mathematics, Geometry Standard)
- Use visualization, spatial reasoning, and geometric modeling to solve problems. (NCTM Standard for School Mathematics, Geometry Standard)

Assessment

Assessment Product	Assessment Criteria
Solve various types of projectile motion word problems. Student Resource 1.3.1—Worksheet: Vertical Motion Problems Student Resource 1.3.2—Worksheet: Prep Problems Student Resource 1.3.3—Worksheet: Target Practice	Teacher Resource 1.3.1—Answer Key: Vertical Motion Problems Teacher Resource 1.3.2—Answer Key: Prep Problems Teacher Resource 1.3.3—Answer Key: Target Practice
Projectile Motion Problem Solving "Competitions" Student Resource 1.3.4—Worksheet: Bad Neighbors Student Resource 1.3.5—Worksheet: Mountain Matchup Student Resource 1.3.6—Worksheet: Siege	Teacher Resource 1.3.4—Answer Key: Bad Neighbors Teacher Resource 1.3.5—Answer Key: Mountain Matchup Teacher Resource 1.3.6—Answer Key: Siege

Prerequisites

- Working knowledge of how to solve quadratic equations by completing the square and by factoring
- Working knowledge of how to calculate angles using SOH CAH TOA
- Introduction to projectile motion formulas from Lesson 1.2 (Physics or Principles of Engineering)

Instructional Materials

Teacher Resources

- Teacher Resource 1.3.1—Answer Key: Vertical Motion Problems
- Teacher Resource 1.3.2—Answer Key: Prep Problems
- Teacher Resource 1.3.3—Answer Key: Target Practice

- Teacher Resource 1.3.4—Answer Key: Bad Neighbors
- Teacher Resource 1.3.5—Answer Key: Mountain Matchup
- Teacher Resource 1.3.6—Answer Key: Siege

Student Resources

- Student Resource 1.3.1—Worksheet: Vertical Motion Problems
- Student Resource 1.3.2—Worksheet: Prep Problems
- Student Resource 1.3.3—Worksheet: Target Practice
- Student Resource 1.3.4—Worksheet: Bad Neighbors
- Student Resource 1.3.5—Worksheet: Mountain Matchup
- Student Resource 1.3.6—Worksheet: Siege

Equipment and Supplies

- Computer
- LCD projector
- Mini catapult (optional if no LCD projector)
- Overhead (optional)

Lesson Steps

Step	Min.	Activity
		CLASS PERIOD 1
1	**15**	**Demonstration: Bullseye** You will need a computer and LCD projector for this demonstration. Download the projectile motion applet at http://phet.colorado.edu/new/simulations/sims.php?sim=Projectile_Motion After downloading the applet to your desktop, open the file with an Internet browser. You will need to have a Shockwave plug-in installed for the program to run properly. If you do not have access to an LCD projector, you can do this demo using a mini catapult. The PLTW teacher on your team should be able to provide one for you. Many variables can be adjusted using this simulation. Play around with it prior to class to familiarize yourself with how it works, but in class the only variables students should be concerned with are **angle** and **initial speed**. Mass does not affect the trajectory, and diameter and drag coefficient have an impact only if the air resistance box is checked. For the sake of simplicity, you should leave the air resistance box unchecked. Allow the class to choose an item from the menu to launch. "Fire" the cannon. The object should miss the target if you use the default settings. After the initial miss, allow

Step	Min.	Activity
		students to make suggestions for changing the initial speed, angle, or altitude at which the object is launched. Try their suggestions. Encourage them to adjust only one variable at a time. After you have hit the target, explain to students that in this lesson they will be learning to make calculations that will allow them to hit the target without trial and error. Other, less flashy simulations can be found at http://jersey.uoregon.edu/vlab/Cannon/ http://www.lcse.umn.edu/specs/labs/catapult/index.html http://library.thinkquest.org/27585/lab/sim_pirates.html
2	**20**	**Mini Lecture: Quadratic Formula** Introduce or review the use of the quadratic formula to solve equations as you normally do. Including instruction on how to program a calculator to solve quadratic equations reduces busywork later in the lesson. This activity may take longer than the allotted time if you are introducing this concept for the first time. Remind students that they have encountered quadratic equations in their Physics or Principles of Engineering class during the instruction on projectile motion (Lesson 1.2). Review the variations on the displacement formula below: • Projectile motion is broken up into two independent components: horizontal motion and vertical motion. • The general displacement formula is $d = \frac{1}{2}at^2 + v_0t + d_0$. • In the horizontal direction, acceleration is zero, velocity is constant, and initial distance is (typically) zero. Therefore, if you substitute 0 for *a* and for d_0, the horizontal motion equation is $x = v_0t$. • In the vertical direction, acceleration is due to gravity (–9.8 m/s if using metric units, –32 ft/s if using English units), velocity at a given time is solved using the equation $v = at + v_0$, and initial distance is determined by the height from which you launch the projectile (zero launch is on ground level). Therefore, the vertical displacement formula is $y = \frac{1}{2}at^2 + v_0t + y_0$.
3	**5**	**Model Problem Solving: Projectile Problems** Solve a problem or two for the class by reading the question aloud and model your process for determining how to set up the equation to solve the problem. **Sample 1** An object is launched with an initial velocity of 12.25 meters per second (m/s) from a platform that is 80.85 meters high. How long does it take for the object to hit the ground? **Problem solving** The equation used for an object's height *y* at time *t* is $y = \frac{1}{2}at^2 + v_0t + x_0$. The height when it hits the ground would be zero. Initial velocity (v_0) is given as 12.25 m/s, and the initial height (x_0) is equal to the platform height of 80.85 meters. Acceleration due to gravity is –9.8, so half of that is –4.9. Therefore, this means that our quadratic

Step	Min.	Activity
		equation would be: $0 = -4.9t^2 + 12.25t + 80.85$ or $4.9t^2 - 12.25t - 80.85 = 0$ **Sample 2** An object is launched directly upward at 48 feet per second (ft/s) from a platform that is 60 feet high. When will the object be at 72 feet high? **Problem solving** Let's see, the equation used for an object's height *y* at time *t* is still $\boldsymbol{y = \frac{1}{2}at^2 + v_0t + x_0}$. This time, the initial height is 60 feet, and the initial velocity is 48 ft/s. That means, acceleration will be in feet, –32 ft/s. One other thing, in the previous question, example *y* was 0 because we were interested in when the object hit the ground. This time, we want to know when the object is at 72 feet, so *y* is 72. So, our quadratic equation would be: $72 = -16t^2 + 48t + 60$ or $16t^2 - 48t + 12 = 0$
4	**10**	**Guided Practice: Vertical Motion Problems** Pass out the Vertical Motion Problems worksheet (Student Resource 1.3.1). Give students time in class to continue solving problems on their own while you are available to answer questions. Assign any remaining problems as homework.
		CLASS PERIOD 2
5	**10**	**Mini Lecture: Angles** Introduce or review the use of SOH CAH TOA and arcsin/arccos/arctan to solve for angles as you normally do. This activity may take longer than the allotted time if you are introducing this concept for the first time.
6	**10**	**Modeling: Prep Problems** Pass out the Prep Problems worksheet (Student Resource 1.3.2). Show students how to work through this type of word problem on the overhead or board. Be sure to cover the application of SOH CAH TOA and arctan to solve for angle. Another important skill to cover is drawing a representative diagram to help students visualize information given in the word problems. For examples, see Teacher Resource 1.3.2—Answer Key: Prep Problems.

Step	Min.	Activity
7	**10**	**Guided Practice: Target Practice** Assign students to small groups of two to four. Pass out the Target Practice worksheet (Student Resource 1.3.3). Tell students to solve these problems using the method demonstrated in the Prep Problems you just modeled. Give students time to work on the Target Practice problems in their groups. Remind students to make drawings representing the situation in each of the problems where appropriate. Circulate among the groups as students are working to check for their understanding and help any students who are having difficulty. Alternatively, if you think students are still struggling with these concepts, make transparencies of the worksheets and present them on an overhead so that students can solve them together as a class. If students need original instruction in order to solve for angles, you may need to assign the Target Practice worksheet as homework. If so, be sure to go over the answers carefully the next day because students will need to be able to solve these types of problems quickly in the remaining activities.
8	**5**	**Competition: Bad Neighbors** Match up students into pairs. Each student should be about the same ability level. Pass out the Bad Neighbors worksheet (Student Resource 1.3.4). Assign students to be either Castle A or Castle B (for this activity, the equations will be the same). They will be competing to see which student will be first to find a correct firing solution for their cannon using trig functions and linear equations. This problem has multiple possible solutions. Depending on how students are doing with this activity, you may want to have multiple rounds of this competition. Make new worksheets by changing the range and/or time delay in the scenario.
9	**5**	**Competition: Mountain Matchup** Evaluate the original student pairs and make switches if necessary. Hand out the Mountain Matchup worksheet (Student Resource 1.3.5). Assign students to be either Castle C or Castle D. The next competition requires students to find initial vertical velocity using the sin function, and then to use the quadratic formula to solve first for time at a given height.
10	**10**	**Competition: Siege** Revaluate the student pairs and make switches if necessary. Pass out the Siege worksheet (Student Resource 1.3.6). This next competition is more challenging, requiring students to use the quadratic formula to solve first for time given an initial launch angle and a choice of initial velocity and then asking them to use formulas from the Bad Neighbors challenge to solve for distance.

Extensions

- Include graphing quadratic equations and the associated word problems as components of this lesson. Students can solve for the maximum height and time at maximum height using the formula for vertex, rather than applying the trajectory motion equations presented in this lesson.

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Lesson 1.3

“Qannon” Quadratics

Student Resources

Resource	Description
Student Resource 1.3.1	Worksheet: Vertical Motion Problems
Student Resource 1.3.2	Worksheet: Prep Problems
Student Resource 1.3.3	Worksheet: Target Practice
Student Resource 1.3.4	Worksheet: Bad Neighbors
Student Resource 1.3.5	Worksheet: Mountain Matchup
Student Resource 1.3.6	Worksheet: Siege

Student Resource 1.3.1

Worksheet: Vertical Motion Problems

Directions

Set up and solve the appropriate quadratic equations for each of the following word problems. Include a drawing or diagram representing the problem. Round your answers to the nearest tenth.

1. An object is launched from a 3-meter platform directly upward at 39.2 m/s. For how long is the object at or above a height of 34.36 meters?

2. Starburst Fireworks Company sets up a launching platform for their Fourth of July celebration that is 3 meters high. They want the first shell to explode when it is 297 meters in the air. If the shell is being launched with a vertical velocity of 88.2 meters per second, how long does the fuse need to burn?

3. At graduation, Maria throws her cap into the air at a velocity of 12 feet per second. Maria is 5.5 feet tall. How long does she have if she wants to catch her cap before it hits the ground?

4. You launch a bottle rocket from a launcher on the ground. Its initial vertical velocity is 53.9 meters per second. When will the rocket reach a height of 122.5 meters?

5. At the fair, there is a shooting game. A target appears for a moment 14 feet directly above you every 5 seconds. If you have a dart gun that shoots darts with a velocity of 30 feet per second, how long before the target appears should you shoot your dart gun?

6. You fire an arrow into the air with a vertical velocity of 224 feet per second. Remember, as an object travels upward, it slows down. At the very highest point in its trajectory, its velocity is actually zero, and then the object reverses its direction and begins to fall back down. When will the arrow reach its highest point? You can use the formula $v_f = at + v_0$. Hint: Think about what vf is at the highest point.

7. Okay, now that you've figured out when the arrow is at its highest point, how high did it go? When you shoot an arrow, you hold the bow 5 feet off the ground.

8. The International Space Agency has finally landed a robotic explorer on an extra-solar planet. To demonstrate the crushing weight of gravity on this planet, the camera is aimed at a probe's ejection port that launches a baseball directly upwards at 120 feet per second (ft/s), which is about the top speed of a ball thrown by a professional pitcher. The ejection port is at the top of the probe, about 15 feet off the ground. The force due to gravity on this planet is 100 ft/s^2. Assuming no wind and that the probe can scurry out of the way in time, how long will it take for the ball to hit the ground?

Student Resource 1.3.2

Worksheet: Prep Problems

Directions

In your group, answer the following questions. Include a drawing or diagram representing the problem and show all of your work. Round your answers to the nearest tenth.

1. You are playing putt-putt and you get to the hole where you have to hit the ball through a door. You’ve counted and found out that the door is on a 6-second cycle: closed for 5 seconds and open for 1 second. You estimate that you are 8 meters from the door. If you hit the ball right after the door closes, at what velocity will you have to hit it in order to make the ball go through the door?

2. Your little brother is running down the driveway. He will soon be in range. You’ve figured out how far to throw the water balloon, and in what direction, but you still need to figure out how high to throw it so it hits him at the right time. If you want it to hit him in 4 seconds, what should be the initial vertical velocity?

3. If you throw the balloon with a horizontal velocity of 10 m/s and a vertical velocity of 50 m/s, what is your total initial velocity? (Hint: draw the arrows)

4. How can you solve for the initial angle of the balloon’s trajectory?

Student Resource 1.3.3

Worksheet: Target Practice

Directions

Use the following situations to clarify your understanding of projectile motion. Show all of your work on a separate piece of paper. Round your answers to the nearest tenth.

1. Your enemy's castle is 300 meters away, but is at the same height above sea level as your castle. If it takes 12 seconds to hit your target, figure out the horizontal velocity at which the cannonball will need to be traveling to hit your enemy's castle at the right time. Remember that $d_x = v_{0x}t$.

2. Figure out how fast the cannonball will need to travel initially in a vertical direction to be at the ground when it reaches the castle. You can use the formula $v_f = v_{0y} + at^2$. Remember that at the very top of a trajectory, the velocity is zero, but that it takes only half the total flight time to reach the top of a symmetrical trajectory.

3. Draw the vertical and horizontal velocity vectors. Sum the vectors using the Pythagorean Theorem to determine the total initial velocity.

4. Use the SOH CAH TOA to calculate the angle at which you will need to aim your cannon.

5. If each gram of gunpowder increases the initial velocity by 2 m/s, how many grams of gunpowder will you need to put in the cannon?

6. You order new cannons and decide to test them. If you put in 30 grams of gunpowder, what should be the cannonball's initial velocity?

7. If you fire the cannon at an angle of 30°, what is the initial velocity in just the horizontal direction?

8. What is the initial vertical velocity?

9. If you fire the cannon, how long will it take for the cannonball to reach a height of 36 meters? Remember that $d_y = v_{0y}t + \frac{1}{2}at^2$.

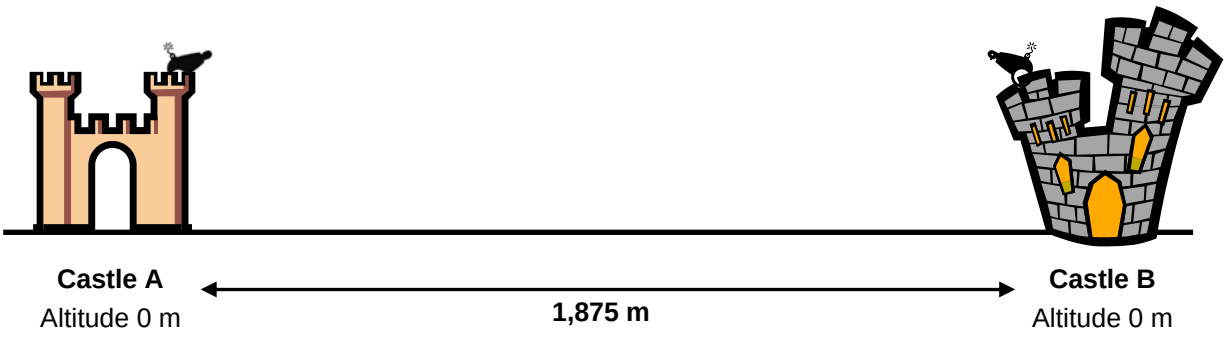
Student Resource 1.3.4

Worksheet: Bad Neighbors

You can’t stand it anymore! The family in the castle next door is driving you crazy: playing loud lute music, having jousts at all hours of the day. This means war!

Every gram of gunpowder you put in the cannon increases the initial velocity of your cannonball by 1 meter per second. You know that it takes a little over 15 seconds for your neighbors to load and fire their own cannon, so your cannonball needs to arrive at its target in 15 seconds (or less). How much gunpowder do you need, and at what initial angle should you aim your cannon in order to hit your target?

The first castle that finds a firing solution wins! Show all of your work and round your final answer to the nearest tenth.

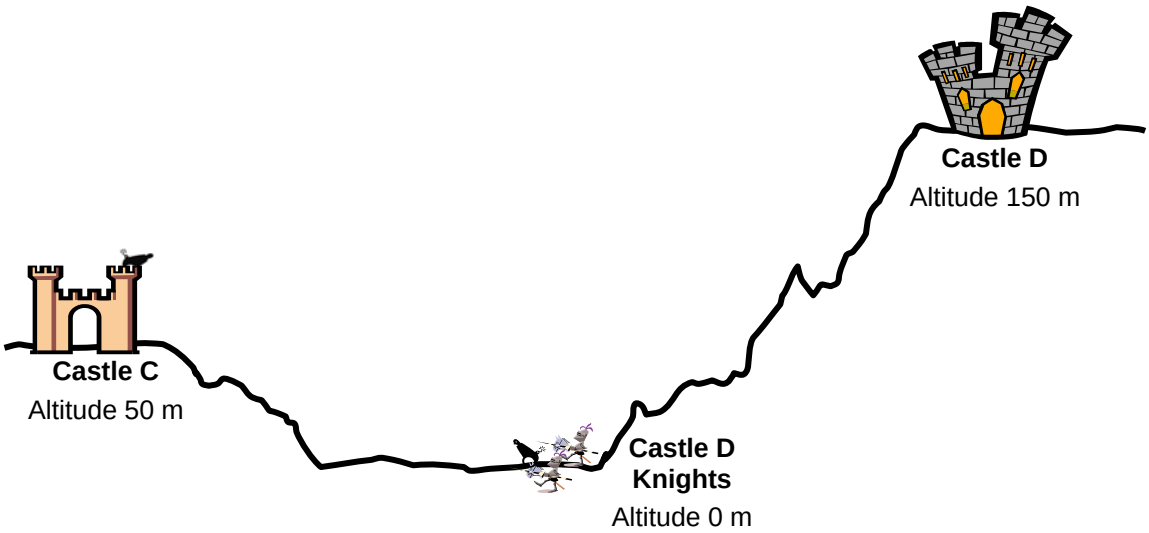


Student Resource 1.3.5

Worksheet: Mountain Matchup

It’s time for a fight in the heights. Unfortunately, you don’t have the high ground, but that doesn’t mean you don’t know what you’re doing. The artillery chief has set the cannons at a 76° angle and tells you to fire with an initial velocity of 56 m/s. **How long will it take your cannonball to reach the castle?**

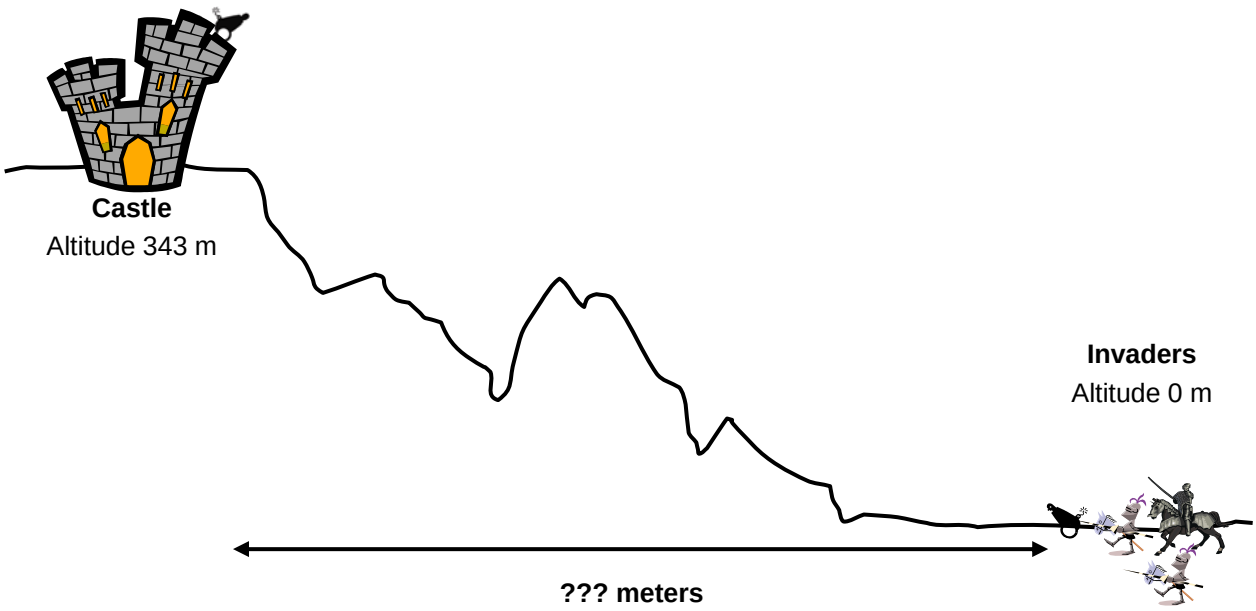
The first side to find a solution wins! Set up all equations and round your answer to the nearest tenth.



Student Resource 1.3.6

Worksheet: Siege

Those jerks, who do they think they are? Don’t they realize you’re in charge? It’s time to get rid of them for good! You have a cannon mounted at a 60° angle. You have gunpowder packs that can launch a cannonball at an initial velocity of either 50 meters per second or 100 meters per second. **How much gunpowder should you use, and at what range should you fire in order to hit the enemy?** Show all of your work in the space below.



AOE POE Integrated Curriculum Unit: Bombs Away

Lesson 1.3

“Qannon” Quadratics

Teacher Resources

Resource	Description
Teacher Resource 1.3.1	Answer Key: Vertical Motion Problems
Teacher Resource 1.3.2	Answer Key: Prep Problems
Teacher Resource 1.3.3	Answer Key: Target Practice
Teacher Resource 1.3.4	Answer Key: Bad Neighbors
Teacher Resource 1.3.5	Answer Key: Mountain Matchup
Teacher Resource 1.3.6	Answer Key: Siege

Teacher Resource 1.3.1

Answer Key: Vertical Motion Problems

Directions

Set up and solve the appropriate quadratic equations for each of the following word problems. Include a drawing or diagram representing the problem. Round your answers to the nearest 10th.

1. An object is launched from a 3-meter platform directly upward at 39.2 m/s. For how long is the object at or above a height of 34.36 meters?

$$d_y = \tfrac{1}{2}at^2 + v_0t + y_0$$
$$34.36 = -4.9t^2 + 39.2t + 3$$
$$4.9t^2 - 39.2t + 31.36 = 0$$
$$t = 7.1 \text{ s or } 0.9 \text{ s}$$

2. Starburst Fireworks Company sets up a launching platform for their Fourth of July celebration that is 3 meters high. They want the first shell to explode when it is 297 meters in the air. If the shell is being launched with a vertical velocity of 88.2 meters per second, how long does the fuse need to burn?

$$d_y = \tfrac{1}{2}at^2 + v_0t + y_0$$
$$395 = -4.9t^2 + 88.2t + 3$$
$$4.9t^2 - 88.2t + 392 = 0$$
$$t = 10 \text{ s or } 8 \text{ s}$$

3. At graduation, Maria throws her cap into the air at a velocity of 12 feet per second. Maria is 5.5 feet tall. How long does she have if she wants to catch her cap before it hits the ground?

$$d_y = \tfrac{1}{2}at^2 + v_0t + y_0$$
$$0 = -16t^2 + 12t + 5.5$$
$$16t^2 - 12t - 5.5 = 0$$
$$t = 1.1 \text{ s}$$

4. You launch a bottle rocket from a launcher on the ground. Its initial vertical velocity is 53.9 meters per second. When will the rocket reach a height of 122.5 meters?

$$d_y = \tfrac{1}{2}at^2 + v_0t + y_0$$
$$122.5 = -4.9t^2 + 48t + 0$$
$$4.9t^2 - 48t + 122.5 = 0$$
$$t = 7.8 \text{ s or } 3.2 \text{ s}$$

5. At the fair, there is a shooting game. A target appears for a moment 14 feet directly above you every 5 seconds. If you have a dart gun that shoots darts with a velocity of 30 feet per second, how long before the target appears should you shoot your dart gun?

$$d_y = \tfrac{1}{2}at^2 + v_0t + y_0$$

$$14 = -16t^2 + 30t + 0$$

$$16t^2 - 30t + 14 = 0$$

$$t = 0.9 \text{ s}$$

6. You fire an arrow into the air with a vertical velocity of 224 feet per second. Remember, as an object travels upward, it slows down. At the very highest point in its trajectory, its velocity is actually zero, and then the object reverses its direction and begins to fall back down. When will the arrow reach its highest point? You can use the formula $\boldsymbol{v_f = at + v_0}$. Hint: Think about what $\boldsymbol{v_f}$ is at the highest point.

$$v_f = at + v_0$$

$$0 = -32t + 224$$

$$t = 7 \text{ s}$$

7. Okay, now that you've figured out when the arrow is at its highest point, how high did it go? When you shoot an arrow, you hold the bow 5 feet off the ground.

$$d_y = \tfrac{1}{2}at^2 + v_0t + y_0$$

$$d_y = -16(7)^2 + (224)(7) + 5$$

$$d_y = 789 \text{ ft}$$

8. The International Space Agency has finally landed a robotic explorer on an extra-solar planet. To demonstrate the crushing weight of gravity on this planet, the camera is aimed at a probe's ejection port that launches a baseball directly upwards at 120 feet per second (ft/s), which is about the top speed of a ball thrown by a professional pitcher. The ejection port is at the top of the probe, about 15 feet off the ground. The force due to gravity on this planet is 100 ft/s^2. Assuming no wind and that the probe can scurry out of the way in time, how long will it take for the ball to hit the ground?

$$y = \tfrac{1}{2}at^2 + v_0t + x_0$$

$$0 = -50t^2 + 120t + 15$$

$$50t^2 - 120t - 15 = 0$$

$$t = 2.5 \text{ s}$$

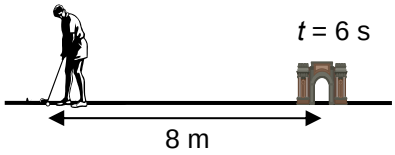
Teacher Resource 1.3.2

Answer Key: Prep Problems

Directions

In your group, answer the following questions. Include a drawing or diagram representing the problem and show all of your work. Round your answers to the nearest 10th.

1. You are playing putt-putt and you get to the hole where you have to hit the ball through a door. You've counted and found out that the door is on a 6-second cycle: closed for 5 seconds and open for 1 second. You estimate that you are 8 meters from the door. If you hit the ball right after the door closes, at what velocity will you have to hit it in order to make the ball go through the door?

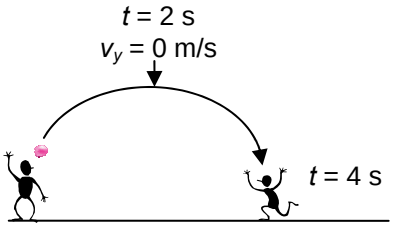


$x = vt$

$8\text{ m} = v \times 5.5\text{ s}$

$v = 1.5\text{ m/s}$

2. Your little brother is running down the driveway. He will soon be in range. You've figured out how far to throw the water balloon, and in what direction, but you still need to figure out how high to throw it so it hits him at the right time. If you want it to hit him in 4 seconds, what should be the initial vertical velocity?



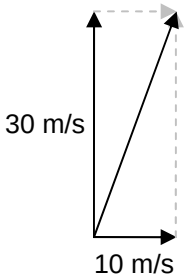
$v_{fy} = v_{0y} + at$

$0\text{ m/s} = v_{0y} + (-9.8\text{ m/s})(2\text{ s})$

$0\text{ m/s} = v_{0y} - 19.6\text{ m/s}^2$

$v_{0y} = 19.6\text{ m/s}^2$

3. If you throw the balloon with a horizontal velocity of 10 m/s and a vertical velocity of 50 m/s, what is your total initial velocity? (Hint: draw the arrows)



$a^2 + b^2 = c^2$

$(10\text{ m/s})^2 + (30\text{ m/s})^2 = c^2$

$c^2 = 100 + 900$

$c^2 = 1000$

$c = 31.6\text{ m/s}$

$\tan\theta = \dfrac{opposite}{adjacent}$

4. How can you solve for the initial angle of the balloon's trajectory?

$\theta = \arctan\left({}^{30}/_{10}\right)$

$\theta = 71.6°$

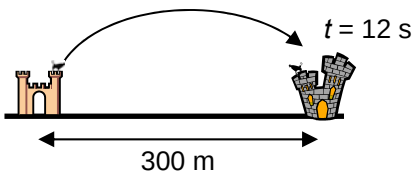
Teacher Resource 1.3.3

Answer Key: Target Practice

Directions

Use the following situations to clarify your understanding of projectile motion. Show all of your work on a separate piece of paper. Round your final answers to the nearest 10th.

1. Your enemy's castle is 300 meters away, but is at the same height above sea level as your castle. If it takes 12 seconds to hit your target, figure out the horizontal velocity at which the cannonball will need to be traveling to hit your enemy's castle at the right time. Remember that $\boldsymbol{d_x = v_{0x}t}$.

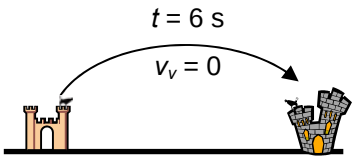


$d_x = v_{0x}t$

$300 \text{ m} = v_{0x} \times 12 \text{ s}$

$v_{0x} = 25 \text{ m/s}$

2. Figure out how fast the cannonball will need to travel initially in a vertical direction to be at the ground when it reaches the castle. You can use the formula $\boldsymbol{v_f = v_{0y} + at}$. Remember that at the very top of a trajectory, the vertical velocity is zero, but that it takes only *half* the total flight time to reach the top of a symmetrical trajectory.

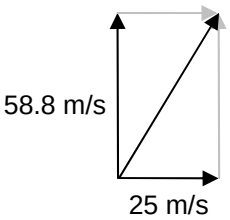


$v_f = v_{0y} + at$

$0 \text{ m/s} = v_{0y} + (-9.8 \text{ m/s}^2)(6 \text{ s})$

$v_{0y} = 58.8 \text{ m/s}$

3. Draw the vertical and horizontal velocity vectors. Sum the vectors using the Pythagorean Theorem to determine the total initial velocity.



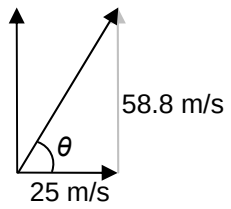
$a^2 + b^2 = v^2$

$(25 \text{ m/s})^2 + (58.8 \text{ m/s})^2 = v^2$

$v^2 = 625 + 3457.44$

$v^2 = 4082.44$

$v = 63.9 \text{ m/s}$



4. Use the SOH CAH TOA to calculate the angle at which you will need to aim your cannon.

$$\tan\theta = \frac{opposite}{adjacent}$$

$\theta = \arctan\left({}^{58.8}/_{25}\right)$

$\theta = 66.9°$

5. If each gram of gunpowder increases the initial velocity by 2 m/s, how many grams of gunpowder will you need to put in the cannon?

v_0 = gunpowder × 2

63.9 m/s = gunpowder × 2

gunpowder = 32 g

6. You order new cannons and decide to test them. If you put in 30 grams of gunpowder, what should the cannonball's initial velocity be?

v_0 = gunpowder × 2

$v_0 = 30 \times 2$

v_0 = 60 m/s

7. If you fire that cannon at an angle of 30°, what is the initial velocity in the horizontal direction only?

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\cos 30 = \frac{v_{0x}}{60}$$

$$0.866 = \frac{v_{0x}}{60}$$

v_{0x} = 52 m/s

8. What is the initial vertical velocity of the cannonball?

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\sin 30 = \frac{v_{0y}}{60}$$

$$0.500 = \frac{v_{0y}}{60}$$

v_{0y} = 30 m/s

9. If you fire the cannon, how long will it take for the cannonball to reach a height of 36 meters?

Remember that $\boldsymbol{d_y = v_{0y}t + \frac{1}{2}at^2}$.

$$d_y = v_{0y}t + \tfrac{1}{2}at^2$$

$$36 = 30t + \tfrac{1}{2}(-9.8)t^2$$

$$4.9\,t^2 - 30t + 36 = 0$$

$$t = 4.5 \text{ s or } 1.6 \text{ s}$$

Teacher Resource 1.3.4

Answer Key: Bad Neighbors

You can't stand it anymore! The castle next door is driving you crazy: playing loud lute music, having jousts at all hours of the day. This means war!

Every gram of gunpowder you put in the cannon increases the initial velocity of your cannonball by 1 meter per second. You know that it takes a little over 15 seconds for your neighbors to load and fire their own cannon, so your cannonball needs to arrive at its target in 15 seconds (or less). How much gunpowder and at what initial angle do you need to aim your cannon in order to hit your target?

The first castle that finds a firing solution wins! Show all of your work and round your final answer to the nearest 10th.

There are many possible solutions because a correct solution can be calculated for any t up to 15 seconds. For this sample solution, 15 seconds is used for t.

Step 1

$d_x = v_{0x}t$

$1875 \text{ m} = v_{0x} \times 15 \text{ s}$

$v_{0x} = 125 \text{ m/s}$

Step 2

$v_f = v_{0y} + at$

$0 \text{ m/s} = v_{0y} + (-9.8 \text{ m/s}^2)(7.5 \text{ s})$

$v_{0y} = 73.5 \text{ m/s}$

Step 3

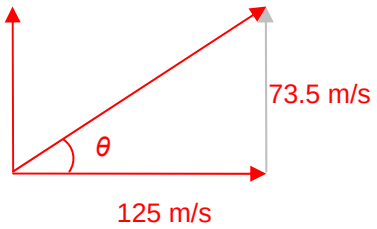
$a^2 + b^2 = v^2$

$(125 \text{ m/s})^2 + (73.5 \text{ m/s})^2 = v^2$

$v^2 = 15{,}625 + 5402.25$

$v^2 = 21{,}027.25$

$v = 145 \text{ m/s}$

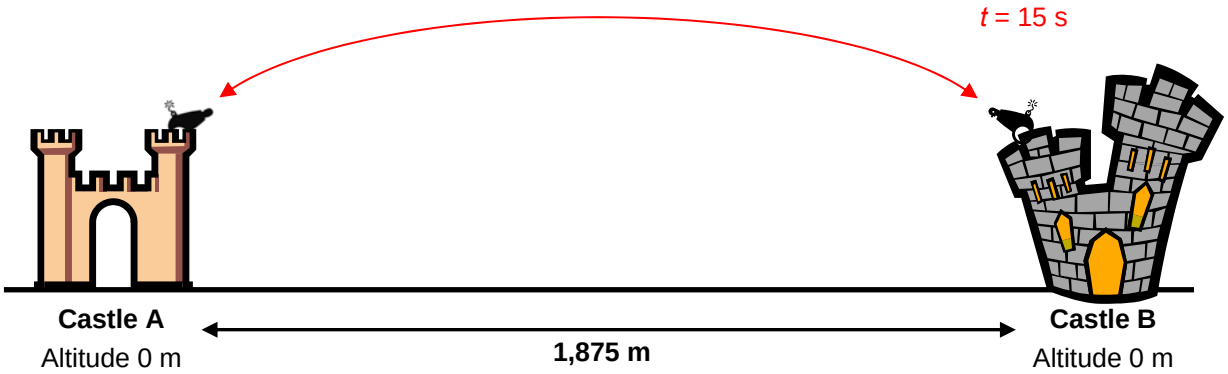


Step 4

$$\tan\theta = \frac{opposite}{adjacent}$$

$\theta = \arctan\left(\frac{73.5}{125}\right)$

$\theta = 30.5°$



Teacher Resource 1.3.5

Answer Key: Mountain Matchup

It's time for a fight in the heights. Unfortunately, you don't have the high ground, but that doesn't mean you don't know what you're doing. The artillery chief has set the cannons at a 76° angle and tells you to fire with an initial velocity of 56 m/s. **How long will it take your cannonball to reach the castle?**

The first side to find a solution wins! Set up all equations and round your answer to the nearest 10th.

CASTLE C

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{v_{0y}}{v_0}$$

$$\sin 76 = \frac{v_{0y}}{56}$$

$$0.97 = \frac{v_{0y}}{56}$$

v_{0y} = 54.3 m/s

$d_y = v_{0y}t + ½\ at^2$

100 m = (54.3 m/s) t + ½ (–9.8 m/s^2) t^2

$4.9t^2 - 54.3t + 100 = 0$

t = 8.7 s or 2.4 s

CASTLE D KNIGHTS

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{v_{0y}}{v_0}$$

$$\sin 76 = \frac{v_{0y}}{56}$$

$$0.97 = \frac{v_{0y}}{56}$$

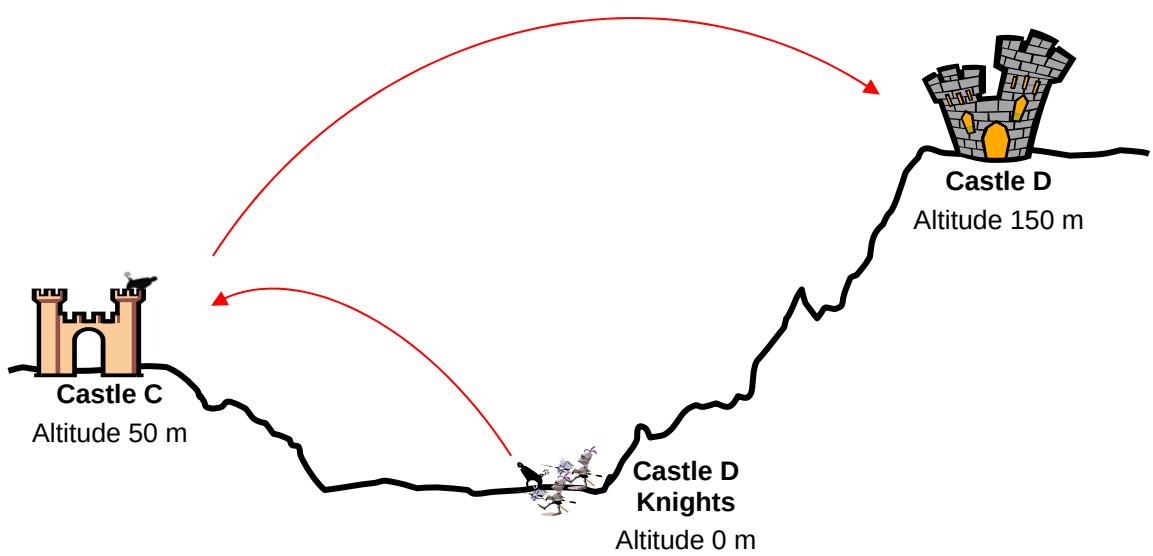
v_{0y} = 54.3 m/s

$d_y = v_{0y}t + ½\ at^2$

50 m = (54.3 m/s) t + ½ (–9.8 m/s^2) t^2

$4.9t^2 - 54.3t + 50 = 0$

t = 10.1 s or 1 s



Teacher Resource 1.3.6

Answer Key: Siege

It's the revolution! Now it's time to do your part. Good thing you've been preparing. You have a cannon mounted at a 60° angle. You have gunpowder packs that can launch a cannonball at an initial velocity of either 50 meters per second or 100 meters per second. **How much gunpowder should you use, and at what range should you fire in order to hit the enemy?** Show all your work in the space below.

There are several two possible solutions to this challenge for both Castle and Invaders. Only one solution is presented for each here.

CASTLE

Gunpowder pack = v_0 = 50 m/s = hypotenuse

θ = 60° using SOH CAH TOA

$$\cos\theta = \frac{adjacent}{hypotenuse} \qquad \sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos 60 = \frac{v_{0x}}{50} \qquad \sin 60 = \frac{v_{0y}}{50}$$

v_{0x} = 0.5 × 50 m/s $\qquad$ v_{0y} = 0.866 × 50 m/s

v_{0x} = 25 m/s $\qquad$ v_{0y} = 43.3 m/s

$d_y = v_{0y}t + ½ at^2$

$-343 \text{ m} = (43.3 \text{ m/s})\, t + ½ (-9.8 \text{ m/s}^2)\, t^2$

$4.9t^2 - 43.3t - 343 = 0$

t = 13.9 s (or –5 s)

$d_x = v_{0x}t$

d_x = 25 m/s × 13.9 s

d_x = 347.5 m

INVADERS

Gunpowder pack = v_0 = 100 m/s = hypotenuse

θ = 60° using SOH CAH TOA

$$\cos\theta = \frac{adjacent}{hypotenuse} \qquad \sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos 60 = \frac{v_{0x}}{100} \qquad \sin 60 = \frac{v_{0y}}{100}$$

v_{0x} = 0.5 × 100 m/s $\qquad$ v_{0y} = 0.866 × 100 m/s

v_{0x} = 50 m/s $\qquad$ v_{0y} = 86.6 m/s

$d_y = v_{0y}t + ½ at^2$

$343 \text{ m} = (86.6 \text{ m/s})\, t + ½ (-9.8 \text{ m/s}^2)\, t^2$

$4.9t^2 - 86.6t + 343 = 0$

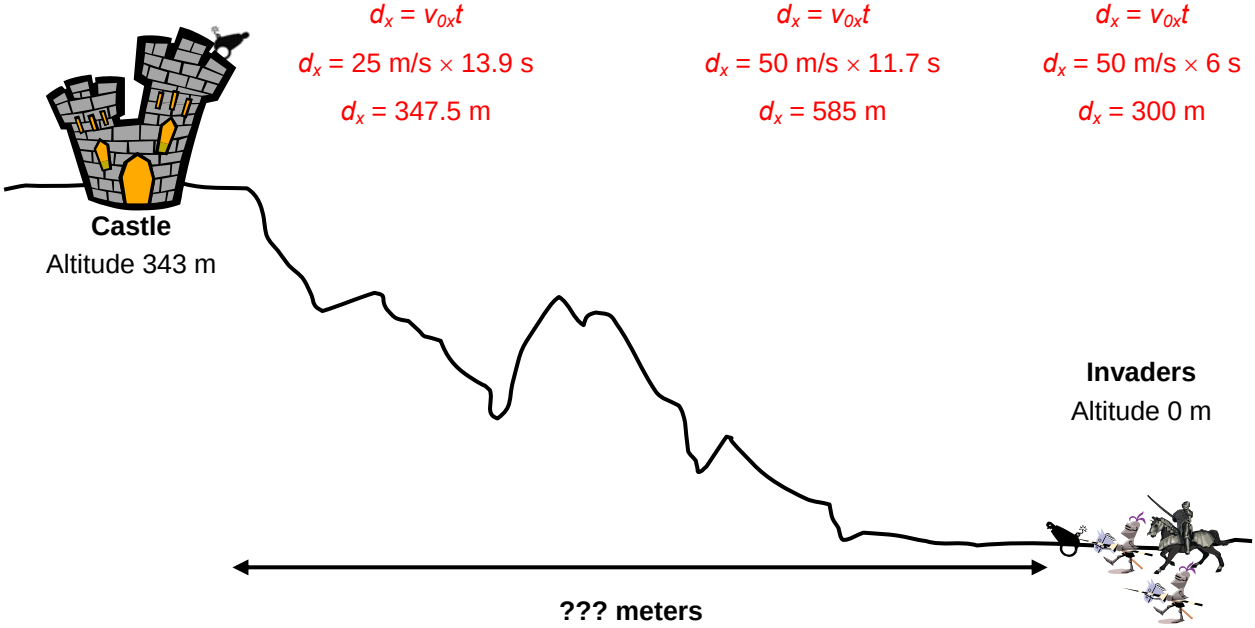
t = 11.7 s or 6 s

$d_x = v_{0x}t$

d_x = 50 m/s × 11.7 s

d_x = 585 m

$d_x = v_{0x}t$

d_x = 50 m/s × 6 s

d_x = 300 m