AOE IED Integrated Unit: Ship Shape

Lesson 2.1

Physical Science

Denser Sensor

In this lesson, students are introduced to the concept of density. Students conduct their own investigation of density by measuring the densities of various materials and then using that information to identify an unknown material. Density is a topic that is typically covered in middle school. However, it can be a difficult concept for younger students to grasp, especially if they do not have a good understanding of ratios. Consult with your students or the science and math teachers from your feeder middle schools to determine how much prior knowledge your students have on the topic. This lesson has been included as a refresher on the concept of density, but it may not be required.

This lesson is expected to take two class periods.

Lesson Framework

Learning Objectives

Each student will:

- Define density as a ratio of mass and volume.
- Distinguish density from mass.
- Calculate the density of a variety of objects.
- Identify objects based on density.
- Predict an object's behavior based on density.

Academic Standards

- Physical Science—Properties and changes of properties in matter
  A substance has characteristic properties, such as density, a boiling point, and solubility, all of which are independent of the amount of the sample. A mixture of substances often can be separated into the original substances using one or more of the characteristic properties. (NRC National Science Education Standards, Standard B)

Assessment

Assessment Product	Assessment Criteria
Density Lab worksheet questions	None provided

Prerequisites

- Introduction to density

Instructional Materials

Teacher Resources

- None

Student Resources

- Student Resource 2.1.1—Worksheet: Pop Prediction
- Student Resource 2.1.2—Worksheet: Density Lab

Equipment and Supplies

- Fish tank (or other large colorless container that will hold water)
- Can of Coca-Cola
- Can of Diet Coke
- Sugar packets and artificial sweetener packets (optional)
- Compressible object, such as a down pillow (optional)
- Also see materials list on Density Lab worksheet (unknown metals are aluminum and nickel)

Lesson Steps

Step	Min.	Activity
		CLASS PERIOD 1

Step	Min.	Activity
1	**25**	**Discrepant Event: Pop Predictions** Have a filled fish tank at the front of the room. Hand out the Pop Prediction worksheet, and then display the Classic Coca-Cola can, and ask students to record their predictions on the worksheet about whether the can will float or sink along with their reasons. Then display the Diet Coke and ask students to make another prediction with reasoning. Put the Diet Coke in the fish tank. Students should observe that the diet soda floats. Ask them to reevaluate their prediction for the regular soda. Put the regular soda in the fish tank; it should sink. --- Alternatively, display the two unmarked soda cans (cover the label with paper). Tell the class that one can contains regular Coca-Cola and one can is Diet Coke. Ask students if there is anyway to tell which one is which without opening and tasting them. Place the two cans in the fish tank. The diet soda should float, while the regular soda should sink. Ask students to predict which soda is which, and to justify their prediction. Take a class vote and ask students to share some of their explanations. Open the sodas and have a taste test to determine which soda was regular and which was diet. Were the class predictions correct? --- Ask students to explain why the two cans did not both either float or sink. What is the difference between regular soda and diet soda? You may want to bring in some sugar packets and some artificial sweetener packets (e.g., Equal, Sweet-n-Low) so students can observe their difference (there is a lot less mass in a single packet of artificial sweetener than there is in a sugar packet). Students should come to realize that although the two soda cans are identical in size, the regular soda has greater mass since it is sweetened with sugar (actually, high-fructose corn syrup). You might have the class compare the actual mass of the each soda using a balance. Note for the class that mass is not the only factor affecting density. Discuss the trick question “Which is heavier: a pound of lead or a pound of feathers?” A pound of feathers takes up a large volume relative to a pound of lead. As a demonstration, you may show a highly compressible object (e.g., a down pillow or a piece of foam). Have a student squash the object into a smaller size. Does it still have the same mass? Does it still have the same volume? Is it more or less dense? Through discussion, students should come to recognize that (1) more mass in the same volume increases density, and (2) the same mass in a smaller volume increases density, but (3) more mass alone does not necessarily increase density.
2	**25**	**Direct Instruction: Density** Introduce or review the formula for density and do a few practice problems for the class. Remind students that they have already covered calculating the volume of a variety of objects in their math class. You may need to review the displacement method for calculating the volume of an irregular solid (including how to read a meniscus) before the lab activity.

Step	Min.	Activity
		CLASS PERIOD 2
3	**40**	**Lab: Density Lab** Before the lab, you will need to prepare the five acetone solutions. Mix the unknown acetone solution to a percentage of your choosing. Unknown metal 1 should be aluminum, and unknown metal 2 should be nickel (or zinc). Density kits are sometimes sold with cubes of various metals. Alternatively, have students use several pieces of metal from chemistry supplies. Pass out the Density Lab worksheet. You may want to demonstrate the initial procedure. Tell students that they will be determining the density of some standard materials and identifying some mystery substances based on density.
4	**10**	**Cleanup: Density Lab** Have students clear up their lab area and begin answering the lab questions. HOMEWORK Students should complete the questions in the Conclusions sections of Parts I and II of the Density Lab worksheet.

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Lesson 2.1

Denser Sensor

Student Resources

Resource	Description
Student Resource 2.1.1	Worksheet: Pop Prediction
Student Resource 2.1.2	Worksheet: Density Lab

Student Resource 2.1.1

Worksheet: Pop Prediction

Question	Will the Coca-Cola float or sink when you put it in water?	Will the Diet Coke float or sink you put it in water?
Prediction		
Explanation		
Observation		
New Explanation		

Student Resource 2.1.2

Worksheet: Density Lab

Density is an important property of matter that is useful in many different scientific fields. Density can be used to identify unknown materials, and it can be used to predict physical behavior. In this lab, you will calculate the density of several known materials and then use that information to identify and predict the behavior of some unknown materials.

Materials

- Distilled water
- Triple-beam balance
- Graduated cylinders
- Graduated cylinder brush
- Liquid soap
- 25% acetone-water solution
- 50% acetone-water solution
- 75% acetone-water solution
- 100% acetone solution
- Unknown acetone solution
- Oil
- Glycerin
- Unknown metal 1
- Unknown metal 2

Part I: Density of Known Liquids

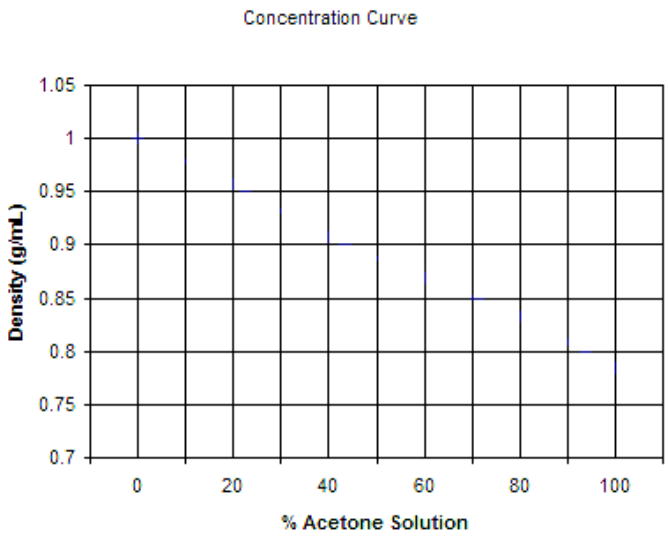
Procedure

1. Measure and record the mass of your graduated cylinder in the data table below.
2. Add distilled water to your graduated cylinder until it is about half full.
3. Read and record the *exact* volume of water in the graduated cylinder in the data table.
4. Measure and record the mass of the graduated cylinder plus water in the data table.
5. Empty and rinse your graduated cylinder.
6. Repeat steps 1–5 with a 25% acetone-water solution.
7. Repeat steps 1–5 with a 50% acetone-water solution.
8. Repeat steps 1–5 with 100% acetone solution.
9. Repeat steps 1–5 with oil (clean your graduated cylinder with soap when you are finished).
10. Repeat steps 1–5 with glycerin (clean your graduated cylinder with soap when you are finished).
11. Calculate and record the mass of each liquid alone.
12. Calculate and record the density of each liquid.

	Step 1	Step 4	Step 7	Step 3	Step 8
Liquid	**Mass of graduated cylinder alone**	**Total mass**	**Mass of liquid alone**	**Volume of liquid**	**Density**
Water					
Acetone 25%					
Acetone 50%					
Acetone 75%					
Acetone 100%					
Oil					
Glycerin					

Conclusions

1. Using the densities of the various acetone solutions, graph a concentration curve on the graph below. Remember that in this case, a 0% acetone solution is the same as water.



2. Among oil, glycerin, and water, which has the highest density? _____

3. Among oil, glycerin, and water, which has the lowest density? _____

4. What typically occurs when oil and water are mixed together?

5. Based on what you know about oil and water, how would you predict a water-and-glycerin mixture to behave?

Part II: Density of Unknown Materials

Procedure

13. Repeat steps 1–5 from Part I with the unknown acetone solution.
14. Calculate and record the density of the unknown acetone solution in the data table below.
15. Measure and record the mass of unknown metal 1.
16. Fill your graduated cylinder with exactly 25 ml of water.
17. Carefully immerse unknown metal 1 in the water without splashing.
18. Read and record the new volume of water.
19. Subtract the original water volume from the new volume to obtain the volume of the unknown metal.
20. Calculate and record the density of the metal.
21. Repeat steps 16–20 for unknown metal 2.

Liquid	Step 1 Mass of graduated cylinder alone	Step 4 Total mass	Step 14 Mass of liquid alone	Step 3 Volume of liquid	Step 14 Density
Acetone Solution					

Liquid	Step 15 Mass of metal	Step 16 Original water volume	Step 18 New water volume	Step 19 Volume of metal	Step 20 Density
Metal #1		25 ml			
Metal #2		25 ml			

Conclusions

1. What percentage of acetone is the unknown solution?

2. Are metal 1 and metal 2 made of the same material? Justify your answer.

3. Based on the chart below, what do you think metal 1 is?

4. Based on the chart below, what do you think metal 2 is?

5. Do you think metal 1 would float or sink in water? Why?

6. If you observed another unknown solid floating at the top of a graduated cylinder filled with oil, what do you think its density might be? How do you know?

AOE IED Integrated Unit: Ship Shape

Lesson 2.1

Denser Sensor

Teacher Resources

There are no Teacher Resources for Lesson 2.1

AOE IED Integrated Unit: Ship Shape

Lesson 2.2

Physical Science

Boat Buoyancy

In this lesson, students are introduced to two key factors in a viable boat design: buoyancy and stability. Students learn to apply Archimedes' principle to predict whether objects will float or sink. Students will also learn how the location of an object's center of mass determines its stability in water. Students will apply their knowledge of these two concepts to construct a neutrally buoyant object.

This lesson is expected to take two class periods.

Lesson Framework

Learning Objectives

Each student will:

- Define and calculate buoyant force on regular and irregular objects.
- Apply Archimedes' principle to answer questions about the buoyancy of objects.
- Accurately predict whether objects will float or sink.
- Explain and apply the importance of center of mass to stable ship design.

Academic Standards

- A substance has characteristic properties, such as density, a boiling point, and solubility, all of which are independent of the amount of the sample. A mixture of substances often can be separated into the original substances using one or more of the characteristic properties. (NRC National Science Education Standards, Standard B)

Assessment

Assessment Product	Assessment Criteria
Buoyancy Problems worksheet	None provided
Neutrally buoyant object	None

Prerequisites

- Working understanding of density

Instructional Materials

Teacher Resources

- Teacher Resource 2.2.1—Reading: Background Information
- Teacher Resource 2.2.2—PowerPoint Presentation: Buoyant Force
- Teacher Resource 2.2.3—PowerPoint Presentation: Stability

Student Resources

- Student Resource 2.2.1—Handout: Loaded Dice
- Student Resource 2.2.2—Worksheet: Buoyancy Problems

Equipment and Supplies

- Heavy can (see Construction Notes, below)
- Water tank
- Pennies
- Scissors
- Tape
- PVC pipe (unevenly weighted; see Construction Notes, below)

Lesson Steps

Step	Min.	Activity
		CLASS PERIOD 1

Step	Min.	Activity
1	5	**Prediction: Float or Sink** Pass the can around the room, and ask students to guess if it will float or sink. Take a vote. Place the can in a tank of water to show students that it does float. **Construction Notes** *Heavy can:* Find an empty can with a tight lid, such as a small coffee can. Partially fill it with sand or another heavy substance until the container only just barely floats.
2	20	**Direct Instruction: Buoyant Force** Using the Buoyant Force PowerPoint presentation as a guide, review Archimedes' principle and introduce the concept of **buoyant force** to the class. Calculate the maximum buoyant force that could act on the can. Students should recognize that since the maximum buoyant force that acts on the can is greater than the can's weight (force due to gravity), the can will float. Students should also recognize that the determination of buoyancy is entirely dependent on the density of the can compared to the density of the fluid.
3	10	**Class Discussion: Challenge Question** Pose the following challenging question to the class: Imagine a small boat in a swimming pool. You are sitting in the boat, holding a massive rock. If you throw the rock overboard into the water, will the level of the water in the swimming pool rise, lower, or stay the same? In fact, the water level should lower when the rock is thrown into the water. This is because when the rock is in the boat, it is displacing a volume of water equal to the weight of the rock. Since a rock is much denser than water, the volume of water being displaced is much larger than the volume of the actual rock. Once the rock is thrown overboard, the volume of water displaced is exactly equal to the volume of the rock.
4	15	**Lab: Loaded Dice** Introduce the second basic consideration of ship design, **stability,** with the following activity. Pass out the Loaded Dice handout, scissors, and tape to each student or group. Have students cut out and assemble the cube. Each group should roll their cube about 30 times, keeping a tally of which side lands facing up. Have each group add up its tallies and combine all results into a set. The class should observe that, on average, the cube landed on each side a similar number of times. Students will continue this activity in the next class period.
		CLASS PERIOD 2

Step	Min.	Activity
5	**15**	**Lab: Loaded Dice, continued** Allow students time to finish recording their rolls of the unloaded die. Then challenge them to make a second cube that lands with the circle side facing up as often as possible. (They do this by taping a penny somewhere on the cube.) Students should then try rolling their cube a few more times. Which groups were successful? Where did they need to tape their penny? (On the heart side)
6	**25**	**Direct Instruction: Stability** What have students observed about the distribution of mass in an object? Using the Stability PowerPoint presentation as a guide, introduce or review the concepts of **center of mass** and **center of buoyancy**. Using the PVC pipe in the water tank, demonstrate the tendency for an object to right itself so that the center of mass is directly below the center of buoyancy. **Construction Notes** *Cylinder (unevenly weighted):* Construct a narrow cylinder whose center of mass is close to one of the ends. An easy way to do this is to use a piece of PVC pipe with PVC caps at either end. Fill one end of the pipe with a dense material (e.g., washers). Stuff the rest of the cylinder with a light material (e.g., bubble wrap) to keep the dense material in place. Seal the two ends of the cylinder with tape of different colors, so students will be able to tell which is the heavy end.
7	**10**	**Building Challenge: Neutral Buoyancy** Challenge students to design a neutrally buoyant cylinder that is stable in the upright position at home this evening. What will the total density of the cylinder have to be? Where will the center of mass need to be? HOMEWORK Students should build their neutrally buoyant object for homework. Have students test their items in class the next day.

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Lesson 2.2

Boat Buoyancy

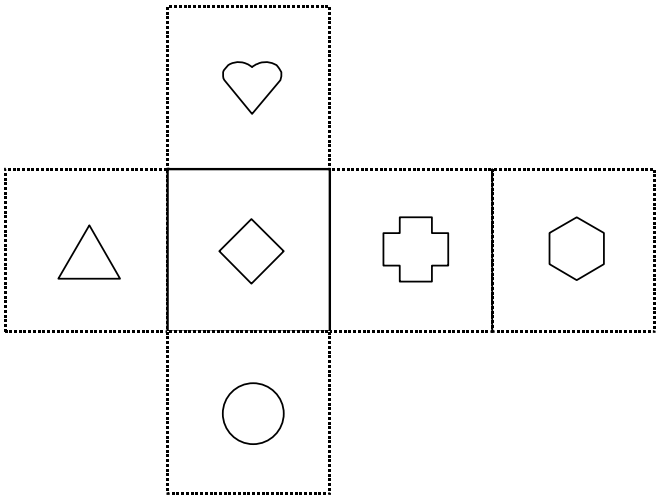
Student Resources

Resource	Description
Student Resource 2.2.1	Handout: Loaded Dice
Student Resource 2.2.2	Worksheet: Buoyancy Problems

Student Resource 2.2.1

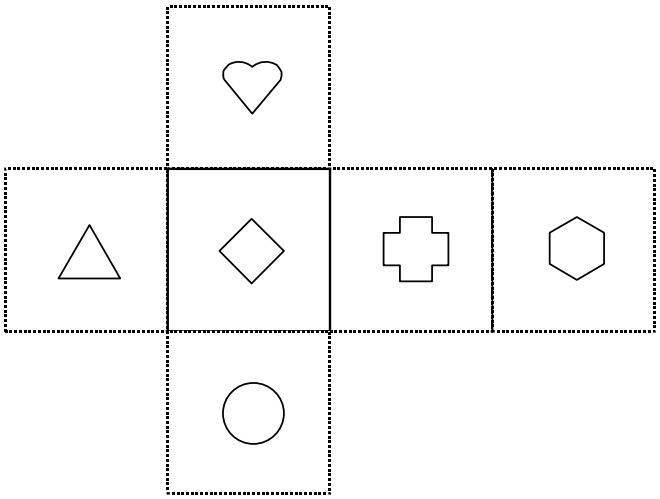
Handout: Loaded Dice

Cut out the template below and fold it into a cube.



Loaded Dice

Cut out the template below and fold it into a cube.



Student Resource 2.2.2

Worksheet: Buoyancy Problems

Directions

Solve each of the following questions on a separate sheet of paper.

1. A nearly weightless but rigid 1 liter milk carton is held submerged under water. What force is required to hold the milk carton under water?

2. A partially filled bottle has a volume of 150 cm^3, and a mass of 125 g. Will it float or sink?

3. A plastic brick has a length of 4 cm, a width of 3 cm, and a height of 3 cm. If the mass of the brick is 30 g, will the brick float or sink?

4. A ball of uniform density is placed in water. If the ball as a density of 0.7 g/cm^3, what percentage of the ball will be underwater?

5. A student has a density of 1050 kg/m^3 (he would sink in ordinary water). His mass is 70 kg.

    a. What is his weight?

    b. What is his volume?

    c. If he were submerged in water, what volume of water would be displaced?

    d. Find the mass of the displaced water.

    e. What is the buoyant force on him?

    f. What is his apparent weight in water?

AOE IED Integrated Unit: Ship Shape

Lesson 2.2

Boat Buoyancy

Teacher Resources

Resource	Description
Teacher Resource 2.2.1	Reading: Background Information
Teacher Resource 2.2.2	PowerPoint Presentation: Buoyant Force
Teacher Resource 2.2.3	PowerPoint Presentation: Stability

Teacher Resource 2.2.1

Reading: Background Information

The design of the seaworthy ship includes consideration of two basic factors: buoyancy and stability. When an object is floating in a fluid, the forces acting upon it are in equilibrium.

$\underbrace{F_1 - F_2}_{\text{Buoyant force } (F_b)} - Mg = 0$

Remember Pascal's law
$P_1 - P_2 = \rho_{fl}\, g\, h$

$F_b = F_1 - F_2$

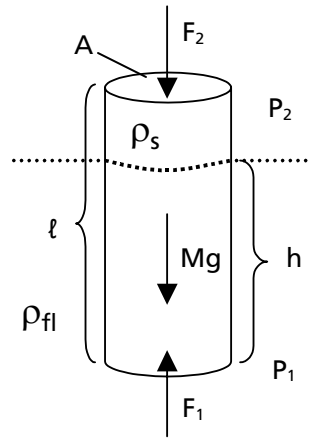
$= AP_1 - AP_2$

$= A\,(P_1 - P_2)$

$= A\,\rho_{fl}\, g\, h$

Equal to the weight of fluid displaced by the solid

This makes sense since $F_1 - F_2 - Mg = 0$ can be simplified to $F_1 - F_2 = Mg$



A	Area
ℓ	Length of solid
h	Height of submerged
M	Mass of solid
g	Gravitational acceleration
P_1	Pressure, fluid
P_2	Pressure, barometric
F_1	Force, fluid
F_2	Force, atmospheric
ρ_s	Density, solid
ρ_{fl}	Density, fluid

The other important factor in building a seaworthy craft is stability. For an object to be stable while floating, the object's center of mass and its center of buoyancy must be on the same vertical axis. If the center of mass and the center of buoyancy are not aligned, the object will tend to rotate until they are aligned. Also, the center of mass should be below the center of buoyancy to maximize stability, otherwise the object will tend to capsize.

Teacher Resource 2.2.2

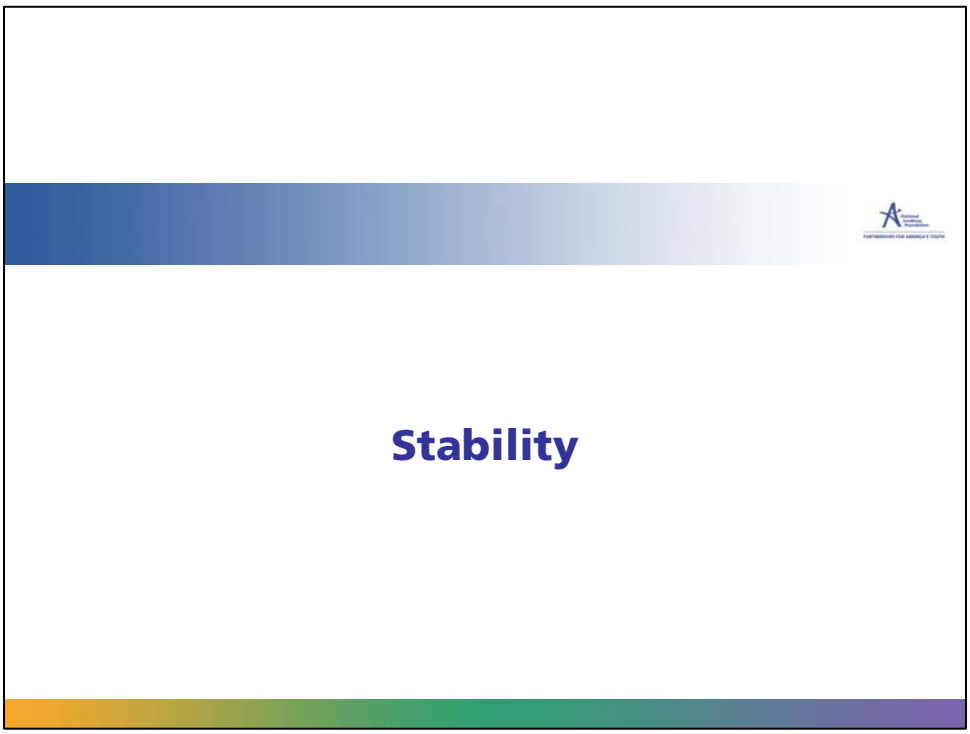
PowerPoint Presentation: Buoyant Force

In order to open the presentation, right-click on the image below and then select Presentation Object/Open from the menu.

Teacher Resource 2.2.3

PowerPoint Presentation: Stability

In order to open the presentation, right-click on the image below and then select Presentation Object/Open from the menu.



AOE IED Integrated Unit: Ship Shape

Lesson 2.3

Biology

A Sinking Feeling

In this lesson, students learn how certain principles important to ship design, such as buoyancy, manifest themselves in the natural world. Students compare the buoyancy control fish display by manipulating their swim bladder organs to the working of submarines.

This lesson is expected to take two class periods.

Lesson Framework

Learning Objectives

Each student will:

- Explain the function of the swim bladder in fish.
- Describe how buoyancy control in fish and in man-made submarine vessels are related.
- Design, build, and control a vessel that can change its buoyancy.
- Apply Boyle's law to solve pressure and volume problems in a marine context.

Academic Standards

- Physical Science—Structure and properties of matter
  The physical properties of compounds reflect the nature of the interactions among its molecules. These interactions are determined by the structure of the molecule, including the constituent atoms and the distances and angles between them.
  Solids, liquids, and gases differ in the distances and angles between molecules or atoms and therefore, the energy that binds them together. In solids the structure is nearly rigid; in liquids molecules or atoms move around each other but do not move apart; and in gases molecules or atoms move almost independently of each other and are mostly far apart. (NRC National Science Education Standards, Standard B)
- Life Science—Behavior of organisms
  Organisms have behavioral responses to internal changes and to external stimuli. Responses to external stimuli can result from interactions with the organism's own species and others, as well as environmental changes; these responses either can be innate or learned. The broad patterns of behavior exhibited by animals have evolved to ensure reproductive success. Animals often live in unpredictable environments, and so their behavior must be flexible enough to deal with uncertainty and change. Plants also respond to stimuli.

Like other aspects of an organism's biology, behaviors have evolved through natural selection. Behaviors often have an adaptive logic when viewed in terms of evolutionary principles. (NRC National Science Education Standards, Standard C)

Assessment

Assessment Product	Assessment Criteria
Working submarine model with variable buoyancy control	None
Swim Bladder Problems worksheet	None provided

Prerequisites

- Working knowledge of density and associated calculations.
- Working knowledge of buoyancy and associated calculations.

Instructional Materials

Teacher Resources

- Teacher Resource 2.3.1—PowerPoint Presentation: Do Fish Need Water Wings?
- Teacher Resource 2.3.2—Reading: Background Information

Student Resources

- Student Resource 2.3.1—Handout: Submarine Challenge
- Student Resource 2.3.2—Worksheet: Swim Bladder Problems

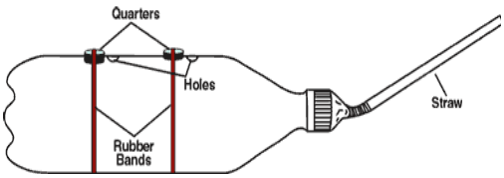
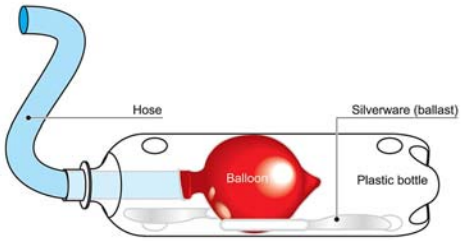
Equipment and Supplies

- Cartesian diver
- Empty 16 or 20 oz plastic soda bottles
- Wide rubber bands
- Pennies
- Aluminum foil
- Flexible straws

- Balloons
- Tape
- Scissors
- Modeling clay
- Large tub of water

Lesson Steps

Step	Min.	Activity
		CLASS PERIOD 1
1	**10**	**Demonstration: Cartesian Diver** Place a Cartesian diver in a 2 liter bottle filled with water. **Construction Notes** You can make a simple Cartesian diver with an eyedropper about half filled with water. Place the eyedropper in the bottle, and it should barely float, vertically, near the top of the bottle. Cap the bottle tightly and demonstrate how the diver rises and sinks when the bottle is squeezed and released. Ask students to hypothesize about what is occurring. Have students think about what effect squeezing the bottle might have on the air bubble inside the Cartesian diver, and how that might relate to what they've learned about density. Explain that the diver is an object suspended in a chamber of water. The diver is buoyant enough to barely float at the top of the chamber. When the chamber is squeezed, the increased pressure compresses the air bubble inside the diver, making the diver neutrally or negatively buoyant. The air in the diver has become pressurized to a small degree, so its volume is decreased. As a result, the air is not displacing enough water to make the diver float, so it sinks.
2	**20**	**Direct Instruction: Buoyancy** Introduce buoyancy as a concept important in the natural world through a lecture on the buoyancy of fish. Use the Do Fish Need Water Wings PowerPoint presentation to cover the swim bladder and how fish use that organ to adjust their own buoyancy.

Step	Min.	Activity
3	**20**	**Lab: Submarine Challenge** Explain to students that undersea vessels (**submarines**) use mechanisms very similar to swim bladders to aid in their propulsion and maneuvering. Pass out the Submarine Challenge handout, and have students design and build their own working submarine based on what they've learned using materials you provide. (Edit the materials list in the handout to reflect the materials you have available. Also, you should probably prepunch holes in the caps using a hammer and large nail.) Alternatively, have students use the procedure at http://www.history.navy.mil/branches/org8-3a.htm. Some possible designs include these:   Students will probably not have enough time to complete this activity in a single class period. Provide a place in the classroom where they can store their designs safely.
		CLASS PERIOD 2
4	**30**	**Lab: Submarine Challenge continued** Continue the submarine challenge where students left off in the last class period. You can extend the challenge by having students use a chemical reaction to increase buoyancy.
5	**20**	**Demonstration: Model Submarines** After students have finished their designs, have them demonstrate their working models. For credit, submarine models should demonstrate the ability to both rise and fall in the water under students' control. HOMEWORK (optional) Pass out the Swim Bladder Problems worksheet, and have students complete it for homework.

AOE IED Integrated Unit: Ship Shape

Lesson 2.3

A Sinking Feeling

Student Resources

Resource	Description
Student Resource 2.3.1	Handout: Submarine Challenge
Student Resource 2.3.2	Worksheet: Swim Bladder Problems

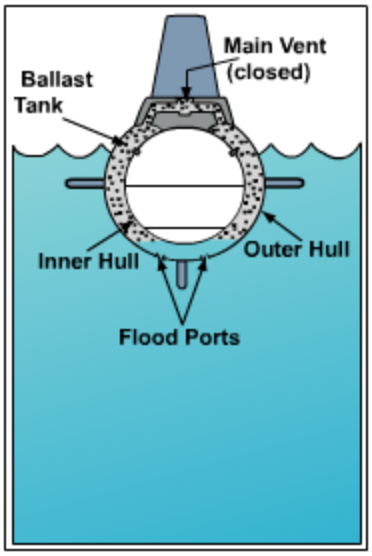
Student Resource 2.3.1

Handout: Submarine Challenge

Submarines work on the same principles as the swim bladders in fish. Buoyancy is controlled by the ballast tanks, which are found between the submarine's inner and outer hulls.

A submarine resting on the surface has **positive buoyancy,** which means it is less dense than the water around it, so it will float. At this time, the ballast tanks are mostly full of air.

To submerge, the submarine must have **negative buoyancy.** Vents on top of the ballast tanks are opened. Seawater coming in through the flood ports forces air out of the vents. Once the submarine ballast tanks are filled with seawater the submarine is denser than the surrounding water, so it will begin to sink. The exact depth can be controlled by adjusting the water-to-air ratio in the ballast tanks. Submerged, the submarine can obtain **neutral buoyancy.** That means the weight of the submarine equals the amount of water it displaces. The submarine will neither rise nor sink in this state.



To make the submarine rise again, compressed air is simply blown into the tanks, forcing the seawater out. The submarine gains positive buoyancy (becomes less dense than the water) and rises.

Materials

- Empty 16 or 20 oz plastic soda bottle
- Wide rubber bands
- Pennies
- Aluminum foil
- Flexible straws
- Balloons
- Tape
- Scissors
- Modeling clay
- Large tub of water

Challenge

Using the soda bottle (as the main hull) and the other materials provided, build a working submarine. Consider the following:

1. Should the sub be positively or negatively buoyant at the start?
2. What will adding air inside do to the sub?
3. How can the density of the total structure be changed easily?

Student Resource 2.3.2

Worksheet: Swim Bladder Problems

Many of the exotic fish in aquariums are collected by fishing with hook and line. Extreme care must be taken when the fishes are brought to the surface, especially if they come from great depths. The atmospheric pressure at the surface is relatively constant and is a standard reference for pressure measurement, referred to as **one atmosphere of pressure** (atm). However, that pressure increases by one atmosphere for each 33 feet of sea water. The pressure affects the gases in the fishes' bodies, decreasing the volume occupied by the gases with each increase in depth. Conversely, as a fish rises in the water column, the decreasing pressure causes the volume of the gases in the body—primarily the swim bladder—to increase. If the rise occurs too rapidly, tissue and organ damage will kill the fish.

The change in pressure and volume can be easily predicted using a basic law of physics. Boyle's law states that the volume and pressure of a gas are inversely related. If P1 = initial pressure, V1 = initial volume, P2 = new pressure, and V2 = new volume, then

$$P1V1 = P2V2$$

For example, if a fish at 66 feet (3 atm) has a swim bladder that occupies 0.25 liters, you can determine the size of the swim bladder at the surface as follows:

$$\begin{aligned} P1 &= 3 \text{ atm} \\ V1 &= 0.25 \text{ liters} \\ P2 &= 1 \text{ atm} \\ P1V1 &= P2V2 \\ V2 &= P1V1/P2 \\ &= (3 \text{ atm})(0.251)/(1 \text{ atm}) \\ &= 0.751 \end{aligned}$$

Questions

1. A large red snapper hooked at 198 feet has a surface swim bladder size of 0.94 liters. How big was the swim bladder at the time it was hooked?

2. A grouper had a swim bladder size of 0.15 liters when hooked at 99 feet. How big was the swim bladder at the surface?

3. If a jewelfish had a swim bladder size of 0.17 liters at the point it was hooked and 1.02 liters at the surface. How deep was the fish when it was hooked?

4. A gray tiger fish hooked at 175 feet has a surface swim bladder size of 0.66 liters. How big was the swim bladder at the time it was hooked?

5. An amberjack had a swim bladder size of 0.2 liters when hooked at 110 feet. How big was the swim bladder at the surface?

6. A barracuda had a swim bladder size of 0.15 liters at the point it was hooked and 1.1 liters at the surface. How deep was the fish when it was hooked?

AOE IED Integrated Unit: Ship Shape

Lesson 2.3

A Sinking Feeling

Teacher Resources

Resource	Description
Teacher Resource 2.3.1.	PowerPoint Presentation: Do Fish Need Water Wings?
Teacher Resource 2.3.2	Reading: Background Information

Teacher Resource 2.3.1

PowerPoint Presentation: Do Fish Need Water Wings?

In order to open the presentation, right-click on the image below and then select Presentation Object/Open from the menu.

Teacher Resource 2.3.1

Reading: Background Information

For more information, see http://www.bio.davidson.edu/Courses/anphys/2000/Martin/introduction.html.

The **swim bladder** is an organ found in some types of fish (most bony fish) that allows fish to control their buoyancy. This organ (also known as the **gas bladder** or **air bladder**) is a flexible, gas-filled sac found in the dorsal portion of a fish's body cavity. Some fish can fill their swim bladder by gulping air at the surface. However, many fish require a special organ called the **gas gland.** When additional gas is needed in the swim bladder, the gas gland excretes lactic acid, which deoxygenates hemoglobin in the blood. The free oxygen is used to fill the swim bladder.

In general, the swim bladder is used by the fish to maintain neutral buoyancy, but it can also be used to assist the fish in ascending or descending in water, with a minimal expense of energy for swimming. As a fish swims to water of different depths, pressure and salinity change, affecting the fish's buoyancy. If a fish swims into water and becomes **positively buoyant** (floats upward), it can release gas from the swim bladder to reduce buoyancy. When the fish swims into water and begins to sink, gas can be taken into the swim bladder to increase buoyancy. Species of fish that do not possess a swim bladder sink to the bottom if they stop swimming.

AOE IED Integrated Unit: Ship Shape

Lesson 2.4

Geometry

3-D Solids

In this lesson, students review and extend their knowledge of area and volume. They explore various three-dimensional solids by constructing two-dimensional nets and then deriving the formula for surface area based on their knowledge about the area of standard geometric shapes. Students are also introduced to additional volume formulas and apply those formulas in calculations for various hull shapes.

This lesson is expected to take two class periods.

Lesson Framework

Learning Objectives

Each student will:

- Calculate the surface area of regular geometric solids.
- Calculate the volume of regular geometric solids.
- Apply volume formulas and density formulas to determine the buoyancy of objects.

Academic Standards

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
  Use visualization, spatial reasoning, and geometric modeling to solve problems. (NCTM Standards for School Mathematics, Geometry)
- Use mathematical models to represent and understand quantitative relationships.
  Analyze change in various contexts. (NCTM Standards for School Mathematics, Algebra)

Assessment

Assessment Product	Assessment Criteria
Exit Ticket: Volume Problem	None provided
Boat Size worksheet problems	None provided

Prerequisites

- Working knowledge of how to solve for area of standard geometric shapes.
- Working understanding of volume and area definitions.
- Prior experience with volume calculations.

Instructional Materials

Teacher Resources

- Teacher Resource 2.4.1—Reading: Background Information

Student Resources

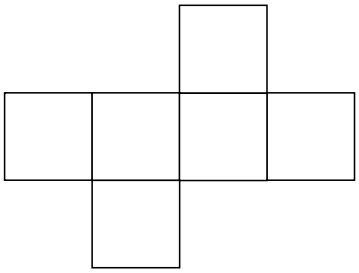
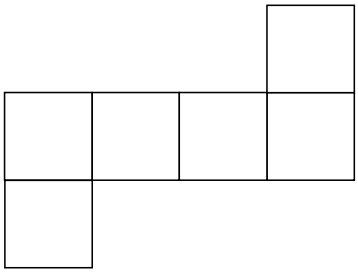
- Student Resource 2.4.1—Worksheet: Boat Geometry
- Student Resource 2.4.2—Worksheet: Boat Size

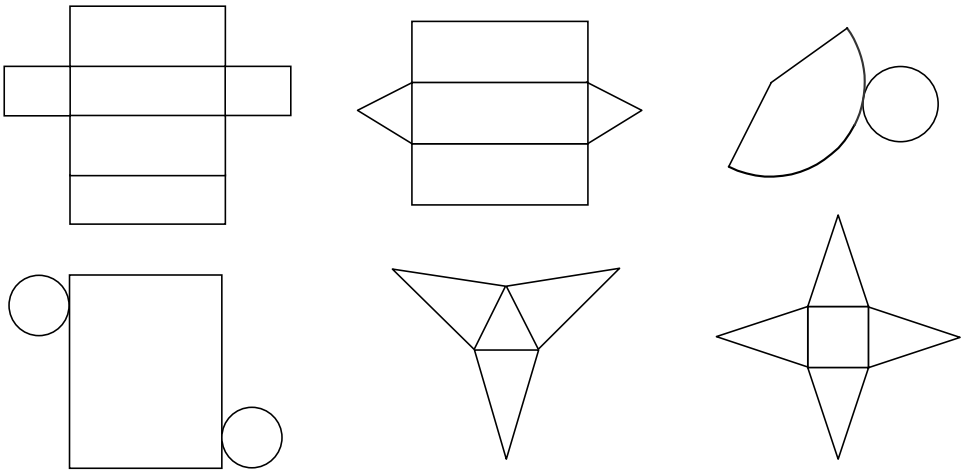
Equipment and Supplies

- Graph paper

Lesson Steps

Step	Min.	Activity
		CLASS PERIOD 1

Step	Min.	Activity
1	10	**Class Discussion: Geometric Shapes** Pass out the Boat Geometry worksheet. Ask students where they see geometry in boats and ships. Possible answers include sail shape, bow shapes, hull cross-sections, and so on. Ask students to consider why these items come in particular shapes. Is it a result of their function; is it just for appearance?
2	20	**Direct Instruction: Volume and Surface Area** Define **surface area** and **volume.** Then define **prism** and **cylinder** for the class. A deck of cards is a useful tool for explaining right and oblique prisms. Each card in the stack is congruent to the base, just as any slice parallel to the base of a prism is congruent to the base regardless of whether the prism is right or oblique. Show examples of prisms with different bases and point out that lateral faces are always parallelograms.
3	20	**Model Building: 3-D Solids** Display a cube to the class. Explain that a net is an unfolded two-dimensional representation of a 3-D figure. Students will have recently seen the net of a cube in Lesson 2.2, when they made dice. You may draw a couple of other cube nets on the board as examples.   Break the class into 6 or 12 groups and pass out graph paper. Display a range of geometric solids to the class: rectangular prism, triangular prism, cone, cylinder, pyramid with a triangular base, and pyramid with a rectangular base. Assign each group of students a solid. Working individually or as groups, have students make a nets of their assigned solid. Continue this activity in the next period.
		CLASS PERIOD 2

Step	Min.	Activity
4	**20**	**Model Building: 3-D Solids** Students should continue making nets of their assigned solid. Possible solutions include:  When each student has completed an accurate net, reorganize the students so that every group has at least one example of each type of geometric solid. In groups, have all the students derive a surface area formula for of each the solids.
5	**20**	**Class Discussion: Boat Hulls** Remind students that they have been studying density and buoyancy in their science and engineering classes. In order for a boat to float, its density needs to be less than that of water. In order to calculate density, they need to know the hull's volume. Boat hulls come in many different shapes and sizes, but for the sake of simplicity, this unit will focus on hulls that are combinations of regular geometric solids. Introduce the formulas for volume, and do several practice problems together as a class.
6	**10**	**Exit Ticket: Volume Problem** As their exit ticket, have students draw and label the measurements of a 3-D solid that has a volume of 240 cm^3 on a note card. HOMEWORK Pass out the Boat Size worksheet, and have students complete it for homework. Students should be familiar with orthographic drawing from their engineering class. Typical orthographic drawings include views of an object from the front, top, and right side.

AOE IED Integrated Unit: Ship Shape

Lesson 2.4

3-D Solids

Student Resources

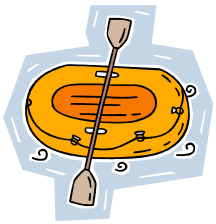
Resource	Description
Student Resource 2.4.1	Handout: Boat Geometry
Student Resource 2.4.2	Handout: Boat Size

Student Resource 2.4.1

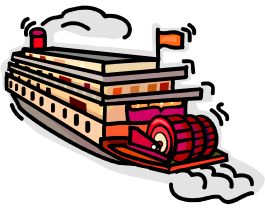
Worksheet: Boat Geometry

Geometry is the foundation behind the design of many real-world objects. How geometry is considered in designing something often affects what it looks like and how it works. Geometry can be seen in a variety of different contexts, including the design of watercraft. Take a look at the vessels pictured here. Identify as many geometric shapes as you can, and list them in the space below.

A



B



C

D

E

What are some advantages of the geometric shapes you see? Try to identify at least one way that the geometric shape in each boat affects how it looks and works.

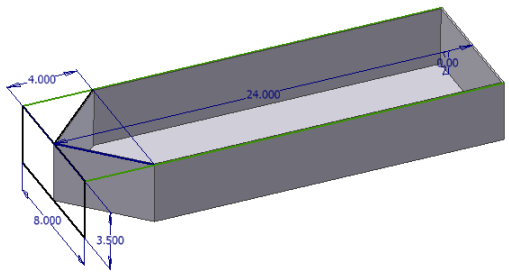
Boat	Geometric shape, and how shape affects appearance	Geometric shape, and how shape affects function
A		
B		
C		
D		
E		

Student Resource 2.4.2

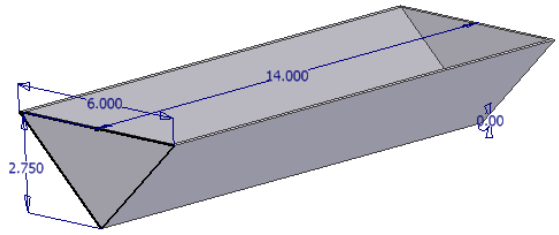
Worksheet: Boat Size

Find the total volume and outer surface area of each of the following hulls. Show all work on a separate piece of paper.

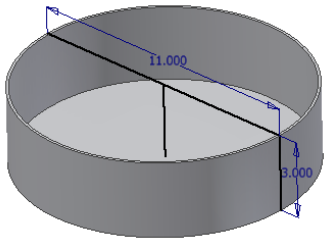
1.



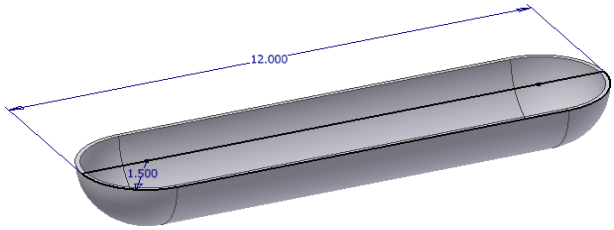
2.



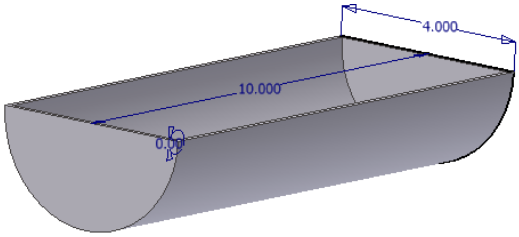
3.



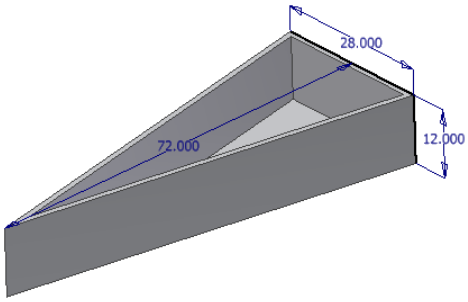
4.



5.



6.



7. Design four additional hulls of different sizes and shapes that have the same volume as boat 6. Submit orthographic drawings with appropriate measurements for each design.

AOE IED Integrated Unit: Ship Shape

Lesson 2.4

3-D Solids

Teacher Resources

Resource	Description
Teacher Resource 2.4.1	Reading: Background Information

Teacher Resource 2.4.1

Reading: Background Information

Volume calculations are extremely important when designing boats and ships. Boats and ships float because the mass of the water that the hull displaces is greater than the total mass of the vessel. In other words, the density of the vessel (the ratio of mass to volume) is less than that of the surrounding water. Fresh water has a density of 1.00 kg/liter. Seawater has an average density of 1.26 kg/liter.

Actual *ship* hulls are curved and tapered to reduce drag when the ship is motion. In professional ship design, calculus is required to determine the volume of a ship's hull. For this unit, students will generally be building hulls with regular geometric shapes.

AOE IED Integrated Unit: Ship Shape

Lesson 2.5
Geometry/Algebra
Cargo Ship Challenge

In this lesson, students are given some specific design constraints for designing a very simple boat to carry cargo. Students must apply the quadratic formula to minimize material waste in their design and must apply their knowledge of buoyancy and density to ensure that their boat will carry the required load.

This lesson is expected to take two class periods.

Lesson Framework

Learning Objectives

Each student will:

- Apply the quadratic and buoyant force formulas in designing a small boat to specific design constraints.
- Predict how great a load the boat is capable of carrying.
- Predict how far down in the water the boat will sit when it is empty and when it's loaded.

Academic Standards

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
  Use visualization, spatial reasoning, and geometric modeling to solve problems. (NCTM Standards for School Mathematics, Geometry)
- Use mathematical models to represent and understand quantitative relationships.
  Analyze change in various contexts. (NCTM Standards for School Mathematics, Algebra)

Assessment

Assessment Product	Assessment Criteria
Boat that conforms to design specifications	None provided

Prerequisites

- Working knowledge of volume formulas
- Working knowledge of solving quadratic equations
- Working knowledge of calculating buoyant force in water

Instructional Materials

Teacher Resources

- Teacher Resource 2.5.1—Reading: Background Information
- Teacher Resource 2.5.2—Answer Key: Lifeboat Model

Student Resources

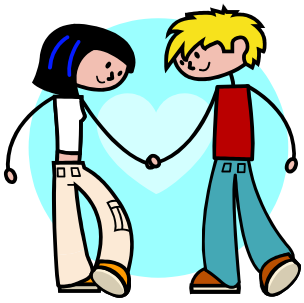
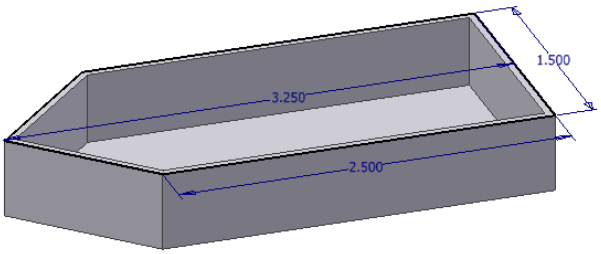
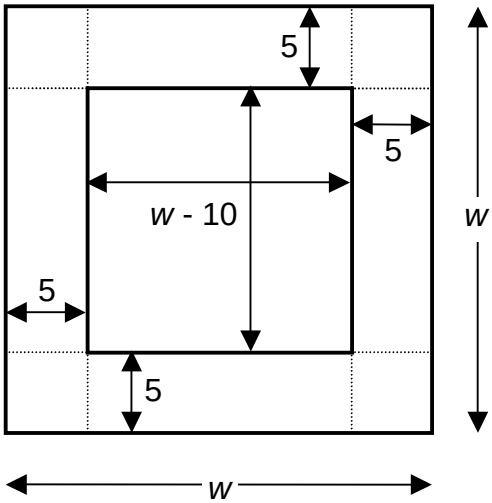
- Student Resource 2.5.1—Handout: Basic Cargo Ship Challenge
- Student Resource 2.5.2—Worksheet: Lifeboat Model

Equipment and Supplies

- Cardstock for building boats
- Index cards
- Tape
- Scissors
- Pennies (several rolls)
- Metric rulers
- Digital balance (kitchen scale or borrow one from Science department)
- Clear plastic container with water for boat testing

Lesson Steps

Step	Min.	Activity
		CLASS PERIOD 1

Step	Min.	Activity
1	5	**Problem-Solving: Boat Problem** Post the problem below on the board. Remind students they will need to apply the content and formulas they learned in Lessons 2.1 and 2.2. (You may also need to remind students that a boat's **draft** is the vertical distance between the water line and the bottom of the boat's hull.) Anthony has a mass of 65 kg. Maria has a mass of 55 kg. The boat below has a mass of 800 kg. The dimensions shown are in meters. The sides of the boat are 0.5 m high. What will the draft of the boat be if Anthony and Maria are aboard?   Work through the problem as a class, after everyone has had a chance to solve it on their own. (The draft should be 0.21 m.)
2	10	**Direct Instruction: Quadratic Equation** Review the quadratic equation for students. The quadratic equation can be useful when designing geometric objects with specific parameters. Say you want to design a square box with a volume of 250 cm^3 and a height of 5 cm. What size is the initial cardboard piece? The unknown is the length of side *w*. In order to get 5 cm sides, 5 by 5 squares will need to be cut out of each corner. When those sides are folded up, both the length and width of the bottom of the box will be $w - 5 - 5$, or $w - 10$. Students should know that volume is length × width × height, so  $(w - 10)(w - 10)(5) = 250$ $(w - 10)(w - 10) = 50$ $w^2 - 20w + 100 = 50$ $w^2 - 20w + 50 = 0$ Applying the quadratic equation gives you $w = \frac{20 \pm \sqrt{400 - 200}}{2} = \frac{20 \pm \sqrt{200}}{2} = \frac{20 \pm 2\sqrt{50}}{2} = 10 \pm \sqrt{50}$

Step	Min.	Activity
3	**35**	**Model Building: Cargo Ship Challenge** Pass out the Cargo Ship Challenge handout. Explain to students that in this lesson they will be combining their knowledge from the unit's math and science lessons to design a small cardboard boat to carry pennies as cargo. The challenge of this lesson is to design a boat within specific size constraints and with the minimum amount of materials wasted when building it. Assign each student slightly different design parameters so that each boat will be slightly different in size. Do not allow students to test their boats until the final presentation.
		CLASS PERIOD 2
4	**45**	**Presentations: Boat Testing** After students have finished building their boats, gather the class for the launching. To save time, have each student make a placard with just the name of the boat, its anticipated maximum capacity, and its cost. Test the boats in rapid succession at their anticipated maximum capacity. If the boat sinks, it should be disqualified. You may wish to allow students to add more cargo onto their boats, but this has the potential to add a lot of time to presentations.
5	**5**	**Reflection: Boat Redesign** Announce the winner of the Cargo Ship Challenge and have students write a short summary of their boat design, including a proof of the boat's buoyancy, the calculations determining maximum load or use of building material, and technical diagrams of the boat design with measurements.

AOE IED Integrated Unit: Ship Shape

Lesson 2.5

Cargo Ship Challenge

Student Resources

Resource	Description
Student Resource 2.5.1	Handout: Basic Cargo Ship Challenge
Student Resource 2.5.2	Worksheet: Lifeboat Model

Student Resource 2.5.1

Handout: Basic Cargo Ship Challenge

Narrative

When building large structures, it is important to minimize waste in construction. The profit margin on a project is reduced when purchased materials go unused or are discarded. Too much waste can turn a great project into a huge loss.

Design Statement

Waste occurs as a result of careless construction, but it can also be caused by a sloppy design. To ensure that waste is minimized, your job is to create the specifications for a boat that fulfills the design constraints while simultaneously minimizing unused material.

- Calculate the exact size of original cardstock needed to build the boat.
- Calculate how many pennies your boat will hold.
- Calculate how deep the draft of the boat will be when fully loaded.
- Build the boat.

Constraints

1. Primary building material must be cardstock.
2. Cardstock must be “purchased” from the cardstock seller (your teacher). The seller will cut your desired piece(s) from a larger piece, but will only make two cuts for free.
3. Boat must have a total volume of _____.
4. Boat sides must be _____ tall.

Student Resource 2.5.2

Worksheet: Lifeboat Model

Scale Drawing

Using the Lifeboat Design Brief from Lesson 1.3 (and chapter 50 of *Life of Pi*, if necessary), create three view (top, side, and front) scale drawings of the lifeboat on graph paper. Be sure to include all relevant measurements and include the scale you are using (e.g., 1 foot = ¼ inch or 2 feet = ½ inch, etc.).

Volume Calculations

1. What is the volume of the interior space of the lifeboat (bench level and below)? Show your calculations.

2. What is the combined volume of the lifeboat's four airtight buoyancy tanks? You may assume the tanks have straight sides.

3. Design a new lifeboat with the same interior volume as Pi's boat. All original constraints are the same except either:
    - Length is 10 feet greater than width; ***or***
    - Length is 20 feet greater than width

New Scale Drawing

Make a three view scale drawing of your new lifeboat with all relevant measurements. Be sure to include the scale you are using.

Scale Models

Construct a scale model of both Pi's original lifeboat ***and*** your new lifeboat design using a scale of your choosing.

AOE IED Integrated Unit: Ship Shape

Lesson 2.5

Cargo Ship Challenge

Teacher Resources

Resource	Description
Teacher Resource 2.5.1	Reading: Background Information
Teacher Resource 2.5.2	Answer Key: Lifeboat Model

Teacher Resource 2.5.1

Reading: Background Information

Archimedes' principle states that an object in fluid (floating or submerged) is buoyed up by a force equal to the weight of the displaced fluid. Weight is calculated by multiplying mass (M) and the acceleration due to gravity (*g*). If the object is floating, then the buoyant force is equal to the weight of the object. This provides us with the following formula:

$$M_{\text{displaced water}} \times g = M_{\text{boat}} \times g$$

Since g is a constant, it is also true that the mass of water being displaced is equal to the mass of the entire object that is floating. The mass can be measured directly, or it can be calculated by multiplying the density of the item (r) by its volume (V). Therefore, in a floating object

$$M_{\text{water}} = M_{\text{boat}}$$

$$M_{\text{water}} = \rho_{\text{boat}} \times V_{\text{boat}}$$

$$\rho_{\text{water}} \times V_{\text{water}} = M_{\text{boat}}$$

The principle of fluid displacement states that when an object is placed in water, the volume of water it displaces is equal to the volume of that portion of the object that is submerged. This gives us the formula

$$V_{\text{water}} = V_{\text{boat (submerged)}}$$

Combining Archimedes' principle and the principle of fluid displacement allows us to predict how deep in the water a boat will sit (also known as draft, D) when it is empty and when it is loaded. The density of fresh water can be expressed in a variety of ways, depending on how volume is being measured:

$1.0\ g/cm^3$ $1000\ kg/m^3$ 1.0 g/ml 1.0 kg/l

The identity property makes most of these calculations very easy, if the object's dimensions are being measured in centimeters. Students need to be more careful if they are measuring objects in meters.

Teacher Resource 2.5.2

Answer Key: Lifeboat Model

Scale Drawing

Using the Lifeboat Design Brief from Lesson 1.3 (and chapter 50 of *Life of Pi*, if necessary), create three view (top, side, and front) scale drawings of the lifeboat on graph paper. Be sure to include all relevant measurements and include the scale you are using (e.g., 1 foot = ¼ inch or 2 feet = ½ inch, etc.).

Volume Calculations

1. What is the volume of the interior space of the lifeboat (bench level and below)? Show your calculations.

   l x w x h = V

   20ft x 5ft x 2ft = 100 ft^3

2. What is the combined volume of the lifeboat's four airtight buoyancy tanks? You may assume the tanks have straight sides.

   l x w x h = V

   2 (20 x 1.5 x 2) = 120 ft^3

   2 (½ b x h x d) = V

   2 (½ x 8 x 3 x 2) = 48 ft^3

   Total volume = 168 ft^3

3. Design a new lifeboat with the same interior volume as Pi's boat. All original constraints are the same except either:
   - Length is 10 feet greater than width; or 24.14 ft x 4.14 ft x 2 ft
   - Length is 20 feet greater than width 16.18 ft x 6.18 ft x 2 ft

New Scale Drawing

Make a three view scale drawing of your new lifeboat with all relevant measurements. Be sure to include the scale you are using.

Scale Models

Construct a scale model of both Pi's original lifeboat ***and*** your new lifeboat design using a scale of your choosing.